Japanese Journal of Applied Physics
Vol. 47, No. 6, 2008, pp. 5000–5006

Solving the Ternary Quantum-Dot Cellular Automata Logic Gate Problem by Means of Adiabatic Switching

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(Received November 6, 2007; accepted January 9, 2008; published online June 20, 2008)

Quantum-dot cellular automata (QCA) are one of the most promising alternative platforms of the future. Recent years have witnessed the development of basic logic structures as well as more complex processing structures, however most in the realm of binary logic. On the grounds that future platforms should not disregard the advantages of multi-valued logic, Lebar Bajec *et al.* were the first to show that quantum-dot cellular automata can be used for the implementation of ternary logic as well. In their study the ternary AND and OR logic functions proved to be the most troublesome primitive to implement. This research presents a revised solution that is based on adiabatic switching. [DOI: 10.1143/JJAP.47.5000]

KEYWORDS: quantum-dot cellular automaton, Hubbard model, inter-cellular Hartree approximation, adiabatic switching, ternary quantum-dot cellular automaton, ternary logic gates

1. Introduction

The quantum-dot cellular automaton (QCA) is one of the few alternative computing platforms that meet most of the desired reliability and performance criteria for computing platforms of the future.[1–3] Since its introduction by Lent *et al.* at the start of the 1990s the approach has advanced in both the theoretical as well as the technological aspect.[4] However, as many similar visionary ideas even the latter bears the influence of the current development trends. Indeed researchers that are taking advantage of this approach usually limit themselves to binary QCA based structures, with the intent of developing substitutes for the current complementary metal-oxide semiconductor (CMOS) integrated circuits. Due to historical technological limitations this disregards the possibility of employing QCAs as a multi-valued processing platform. This is a serious disadvantage since multi-valued processing allows for greater data storage capabilities as well as faster and more sophisticated processing.[5–7]

The initial work on using the QCA platform for multi-valued processing, more precisely ternary logic, was performed by Lebar Bajec *et al.*[8–10] The authors have advanced the basic binary QCA cell (bQCA cell) presented by Lent *et al.* in such a way that it allows the representation of three logic values. They named the newly introduced QCA cell as the ternary QCA cell (tQCA cell). Furthermore the authors show that the QCA wire and QCA inverter retain their functionality with a simple switch of the basic building block (i.e., the substitution of bQCA cells with tQCA cells promotes the two structures to work in a ternary domain). This, however, is not true for the basic geometry that implements binary conjunction and disjunction, namely the majority gate.[8,9,11] The authors solve this issue by developing a more complex and from the size point of view suboptimal structure.[10] Indeed when compared to the binary counterpart the presented solution more than tripled in size. In addition the new structure, although implementing both ternary conjunction as well as disjunction, does not allow the flexibility of the inputs (i.e., using one of the inputs as the selector of the behaviour of the structure), which is one of the more praised about features of the binary majority gate. The complexity of the proposed structure is one of the principal motivators of this research.

In this article we present a solution that is based on adiabatic switching.[12] The decision for its application originates from the benefits that were presented by researchers working on binary QCAs. The foremost two are increased stability of QCA based structures and simplification of the design of memorizing structures. The approach is based on the quantum-mechanic version of the adiabatic theorem. In this view a quantum-mechanic based model was developed for modelling and simulation of tQCA based structures. Indeed the semi-classical model employed by Lebar Bajec *et al.* due to its simplicity is easy to implement and allows the overall estimation of the behaviour of tQCA based structures, but on the other hand with a limited consideration of the quantum-mechanic properties disables the possibility of the introduction of adiabatic switching.[13] The full range of quantum-mechanic properties is taken into account by a quantum-mechanic model that is based on a Hubbard-type Hamiltonian with Coulomb repulsion. By employing this quantum-mechanic model the article will present a simpler and in view of the inputs more flexible structure that implements ternary conjunction and disjunction.

In §2 we present an overview of the ternary quantum-dot cell, followed by its quantum-mechanic description. In §3 we describe the adiabatic switching approach and its influence on the quantum-mechanic model. Section 4 concludes with the realization of a tQCA based structure that employs adiabatic switching to implement ternary disjunction and conjunction.

2. Overview of the tQCA

A QCA is a planar array of, so called, quantum-dot cells (also named QCA cells). Each QCA cell contains a specific number of charges (typically electrons) and its influence on neighbouring cells is due to Coulomb interaction between its charges and the charges residing on them. Inside a single cell the charges reside only at designated locations, the so called quantum dots. They are able to tunnel between adjacent quantum dots, but tunnelling outside of a cell is impossible. QCA cells operate at energy levels where Coulomb interaction prevails over tunnelling. This means that with specific planar arrays (arrangements) of QCA cells it is

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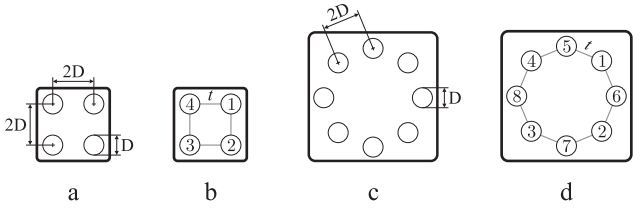

Fig. 1. The geometry of the binary quantum-dot cell presented by Lent *et al.* (a) and the denotation of the quantum dots and the tunnelling paths (b). The geometry of the ternary quantum-dot cell presented by Lebar Bajec *et al.* that was constructed by adding four additional quantum dots to the binary cell (b). The denotation of the quantum dots and the tunnelling paths in the ternary quantum-dot cell (c).

possible to mimic the behaviour of interconnecting wires as well as logic gates and by interconnecting these more complex devices capable of processing can be constructed.

The basic binary QCA, presented by Lent *et al.*, is constructed from bQCA cells and supports the representation of binary information and, as such, also binary processing.[1)] Its following advancement, the ternary QCA, presented by Lebar Bajec *et al.*, consists of tQCA cells and allows the representation of ternary information and enables ternary processing.[8)]

2.1 *The ternary quantum-dot cell*

With the assumption that there are no technological limitations to the number of quantum dots that can be arranged over a single quantum-dot cell the tQCA cell is in its essence an extension of the classic bQCA cell presented by Lent *et al.* [see Figs. 1(a) and 1(c)]. The tQCA cell consists of eight circular quantum dots with diameter $D = 10\,\mathrm{nm}$. The quantum dots are arranged in a circular pattern with radius $D/\sin(\pi/8)$, so that the distance between neighbouring quantum dots equals $2D$. For naming and state representation consistency the denotation of the quantum dots follows the pattern used in the bQCA cell [see Figs. 1(b) and 1(d)]. Furthermore the tQCA cell also contains two electrons, and the same tunnelling properties apply as in the bQCA (i.e., the electrons can tunnel only between adjacent quantum dots and not outside of the cell). The correct inter cellular interaction is possible only if symmetric charge neutralization is assured.[14)] In the case of the tQCA cell the latter is insured by assigning each quantum dot a fixed positive charge of $\rho_+ = e_0/4$, where e_0 is the electron charge.

In an isolated quantum-dot cell the contained electrons due to Coulomb repulsion strive to localize in quantum dots that assure their maximal separation. In the tQCA there are four such arrangements [see Fig. 2(a)]. According to Lebar Bajec *et al.* the arrangement with electrons in quantum dots 2 and 4 is marked as state A, the one with electrons in quantum dots 1 and 3 as B, 5 and 7 as C and 6 and 8 as D. In the absence of external electric fields these four arrangements have exactly the same energy and correspond to the tQCA cell's ground state. This degeneracy manifests as an equally probable localization of the electrons in every dot, which is symbolically represented as in Fig. 2(b). It is said that the tQCA cell is in neutral state. The presence of external influences splits the degeneracy and causes one of the arrangements to become the tQCA cell's ground state.

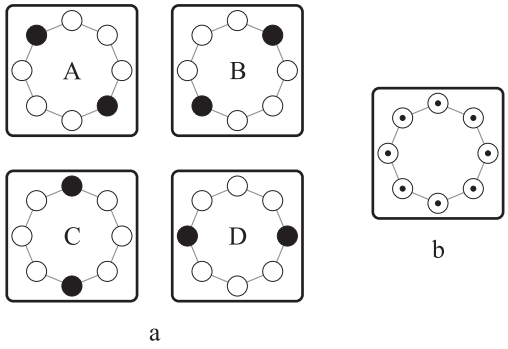

Fig. 2. The four distinct electron arrangements, i.e., the four possible states marked A, B, C, and D of a ternary quantum-dot cell, that correspond to the maximal inter electron separation (a) and the representation of a ternary quantum-dot cell in neutral state (b).

One of the principles that define computing with QCA is ground state computing.[1)] This principle asserts that from the computing point of view the only acceptable state of a QCA cell is its ground state. The four possible electron arrangements of a tQCA cell can thus be interpreted as logic values. State A is interpreted as logic value 0, state B as logic value 1 and state C as logic value 1/2. State D is for reasons that will be explained in the following chapters allowed only as an internal (processing) state.[8–10)]

Another principle, which defines computing with QCA is edge driven computation. It asserts that the input cells, using which data is input into the QCA for processing, are typically situated at the borders of the structure and their states are fixated using external electrostatic fields. Similarly it asserts that the output cells, by means of which the processed data is output from the QCA, are positioned at the borders of the structure as well. Their states are read and interpreted as logic values representing the output of the logic function that is realized by the QCA. The rest of the cells act as internal cells and are the only cells that perform any data transformation.

2.2 *Modelling tQCA*

For the tQCA cell we employ a simple model that is similar to the one used by Lent *et al.* for the bQCA cell, where the quantum dots are represented as sites and the degrees of freedom internal to the quantum dots are ignored.[1)] The corresponding extended Hubbard-type Hamiltonian for the observed cell c is composed of four terms and can be written as:

$$\hat{H}^c = \sum_{i,\sigma}(E_0 + V_i^c)\hat{n}_{i,\sigma} + \sum_{i>j,\sigma} t_{i,j}(\hat{a}_{i,\sigma}^{\dagger}\hat{a}_{j,\sigma} + \hat{a}_{j,\sigma}^{\dagger}\hat{a}_{i,\sigma}) + \sum_i E_Q\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + \sum_{i>j,\sigma,\sigma'} V_Q\frac{\hat{n}_{i,\sigma}\hat{n}_{j,\sigma'}}{r_{i,j}}. \quad (1)$$

The first term of eq. (1) deals with on-site energy, the second term accounts for electron tunnelling between sites, the third term is the on-site charging cost for localizing two electrons of opposite spin at the same site and the last term corresponds to the Coulomb interaction between electrons localized at different sites. The number operator for site i and spin σ is represented by $\hat{n}_{i,\sigma} = \hat{a}_{i,\sigma}^{\dagger}\hat{a}_{i,\sigma}$. Here $\hat{a}_{i,\sigma}$ is the annihilation operator which destroys a particle with spin σ at site i and $\hat{a}_{i,\sigma}^{\dagger}$ is the creation operator for a similar particle.

The values of the physical parameters used are based on a simple, experimentally reasonable model. We take E_0, the ground state energy of a quantum dot holding an electron, to be 1 meV. The potential energy of an electron at site i in the observed cell c due to the existing charges in all other cells of the QCA can be written as:

$$V_i^c = \sum_{k \neq c,j} V_Q \frac{\rho_j^k - \rho_+}{r_{j,i}^{k,c}}, \tag{2}$$

where ρ_j^k is the electron density at site j in cell k, ρ_+ is the fixed positive charge used to maintain charge neutrality and $r_{j,i}^{k,c}$ is the distance between site j in cell k and site i in cell c. The Coulomb coupling strength V_Q was evaluated for a material with a dielectric constant 10 and its value is 143.8 meV. The on-site charging cost $E_Q = V_Q/(D/3)$ is a physically reasonable approximation for the Coulomb energy of two electrons separated by one third of the quantum dot's diameter D. The tunnelling energy $t_{i,j}$ is associated with tunnelling between dots i and j; the selection of its values will be explained in the following chapters.

To find the stationary states of the observed tQCA cell, we solve the time-independent Schrödinger equation

$$\hat{H}^c|\psi_i\rangle = E_i|\psi_i\rangle, \tag{3}$$

where $|\psi_i\rangle$ is the ith eigenstate of the Hamiltonian and E_i is the corresponding eigenvalue. These eigenstates are found using the many-particle site-ket basis for eight sites and two electrons of opposite spins:

$$|\phi_1\rangle \equiv \left|\begin{matrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\end{matrix}\right\rangle,$$
$$|\phi_2\rangle \equiv \left|\begin{matrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{matrix}\right\rangle,$$
$$\vdots$$
$$|\phi_{64}\rangle \equiv \left|\begin{matrix}1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\end{matrix}\right\rangle.$$

In this notation the columns correspond to site (quantum dot) indices, where the first column from the left corresponds to site 1, the second to site 2 and so on. The rows correspond to spin, where the upper row is for spin up and the lower row for spin down. We calculate the Hamiltonian matrix in this basis set by numerically evaluating each matrix element

$$H_{i,j} = \langle\phi_i|\hat{H}^c|\phi_j\rangle, \ i,j = 1,\ldots,64 \tag{4}$$

and finding the eigenvectors of the resulting 64×64 matrix.

The observed cell's ground state, $|\psi_0\rangle$, is in this basis represented as

$$|\psi_0\rangle = \sum_j \psi_j^0|\phi_j\rangle, \tag{5}$$

where $|\phi_j\rangle$ is the jth basis vector and ψ_j^0 is the coefficient of that basis vector, found by direct diagonalization of the Hamiltonian. From the two-electron ground state $|\psi_0\rangle$ we calculate the single-electron density at site i, ρ_i, by finding the expectation value of the number operator for site i:

$$\rho_i = \sum_\sigma \langle\psi_0|\hat{n}_{i,\sigma}|\psi_0\rangle. \tag{6}$$

The single-electron density is then used for quantitative evaluation of the arrangement of electrons in the tQCA cell.

The model described above is exact for a single cell. This is possible because one can explicitly enumerate all possible two-electron states and diagonalize the Hamiltonian in this basis set. Wishing to analyse a QCA based structure composed of a larger number of tQCA cells in the same way would soon reach the boundaries of feasibility. Indeed exact diagonalization methods become intractable as the number of cells and possible many-electron states increase rapidly (e.g., a site-ket basis for a QCA based structure composed of k tQCA cells requires 64^k ket vectors). To overcome this problem when modelling QCA based structures composed of bQCA cells Lent *et al.* proposed a method called Intercellular Hartree Approximation (ICHA).[2] The ground state of the entire system (i.e., the QCA based structure) is found by iteratively solving for the ground state of each cell. A single cell is observed using (3) and the effects of that cell on the potential energies in all other cells are then updated. The intercellular interaction is treated self-consistently using a Hartree approximation.

3. Adiabatic Switching

As stated previously computing with QCA is ground state computing. In view of processing the dynamics of the system is important only in the sense of switching between one ground state and another. This allows for a forthright translation of the system's ground state to the logic solution of the problem that is being solved with it. In this sense switching represents the transition from a cell's state that represents one logic value to a state that represents another. It can be executed in an uncontrolled fashion, also denoted as abrupt switching with dissipative coupling to the environment, or controlled one, known as adiabatic switching.

Adiabatic switching of a QCA based structure adheres to the quantum version of the adiabatic theorem. The latter states that if the Hamiltonian of a system undergoes a gradual change from the initial form H^{i} to a final form H^{f}, and a particle starts in the nth nondegenerate eigenstate of the initial Hamiltonian, it will be carried under the time-dependent Schrödinger equation into the nth nondegenerate eigenstate of H^{f}. In our application of this theorem we transform the Hamiltonian by lowering the inter-dot barriers within the tQCA cell and removing the old input, followed by applying the new input and raising the barriers. If this translations are carried out gradually, the theorem guaranties that the system, which starts in the ground state of the initial Hamiltonian, will be carried smoothly into the ground state of the new Hamiltonian.[12]

The implementation of adiabatic switching in QCA is executed with a cyclic control signal. The signal does not affect the cells directly, but influences on them indirectly through an electric field. The latter acts on the cells' inter-dot barrier heights. It either allows the tunnelling of electrons between adjacent quantum dots and therefore the cells to switch state with respect to their neighbours, or disallows tunnelling and thus locks the cells in their present state. It is desired that the number of cells being controlled by one signal is as large as possible, as it reduces the interconnection complexity necessary for the construction of the required electric fields.

The cyclic control signal comprises four phases of equal length (see Fig. 3).[15] The gradual increase of barrier heights

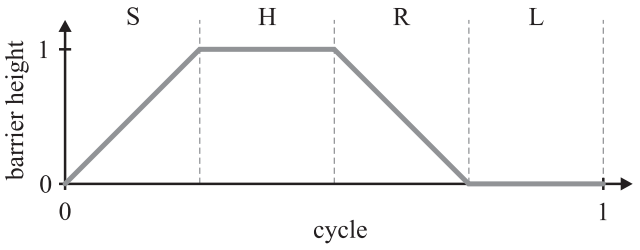

Fig. 3. The cyclic signal that controls the adiabatic switch is composed of four phases, namely: switch (S), hold (H), release (R), and relax (L). In the graph the barrier height is normalized to the interval $[0, 1]$, where value 0 denotes lowered barriers (high probability of the electrons tunnelling between adjacent quantum dots) and value 1 denotes raised barriers (no tunnelling of electrons possible). The four phases are of equal length, thus each lasts $1/4$ of the complete cycle.

is called the switch phase (S) and serves the cells' gradual update of the state with respect to their neighbours. The phase with constant and raised barriers is called the hold phase (H) and is intended for the stabilisation of the cells' states when they are to be transmitted to the neighbours that are in the switch phase (i.e., the cells act as fixated inputs for all other cells). The gradual decrease of the barrier heights and the constant and lowered barriers are called release (R) and relax (L) respectively and support the cells' gradual preparation for a new switch (i.e., the states of the cells gradually transit to a neutral state).

Lent *et al.* advanced the basic premise of adiabatic switching even further. Since the cyclic signal is composed of four phases larger QCA based structures can be decomposed to blocks or subsystems controlled by four phase shifted cyclic signals (see Fig. 4). The obtained structure then shows behaviour similar to a pipeline. Indeed each subsystem is controlled by an independent signal. This allows the decomposition of the computation problem to a number of sub problems and each subsystem can be designated to solve only one. The phase shifted nature of the controlling signals allows the blocks that are in the hold phase to act as inputs for blocks that are in the switch phase. Therefore a subsystem after performing the computation can be designed to lock its state and act as the input for another subsystem. As the transaction is finished the second subsystem can start processing while the first subsystem is ready for processing on new inputs.

As already stated adiabatic switching in its essence controls the tunnelling of electrons between adjacent quantum dots. In the quantum-mechanic model of the tQCA cell (1) this reflects in the parameter t. The Intercellular Hartree Approximation (ICHA) evaluation approach is based on two loops. The outer loop gradually changes the value of parameter t with respect to the chosen cyclic control signal. The inner loop on the other hand performs an iterative evaluation of the ground states of the cells comprising the QCA based structure. Since each change of parameter t is followed by the relaxation process the adiabatic theorem is abided by, as the rate of change in the control signal is slower than the relaxation process. Typical values for parameter t are from the interval $[t_h, t_l]$, where t_h is usually 0 meV and corresponds to the instant when the barrier heights are raised and the tunnelling of electrons impossible. The value of t_l is, on the other hand,

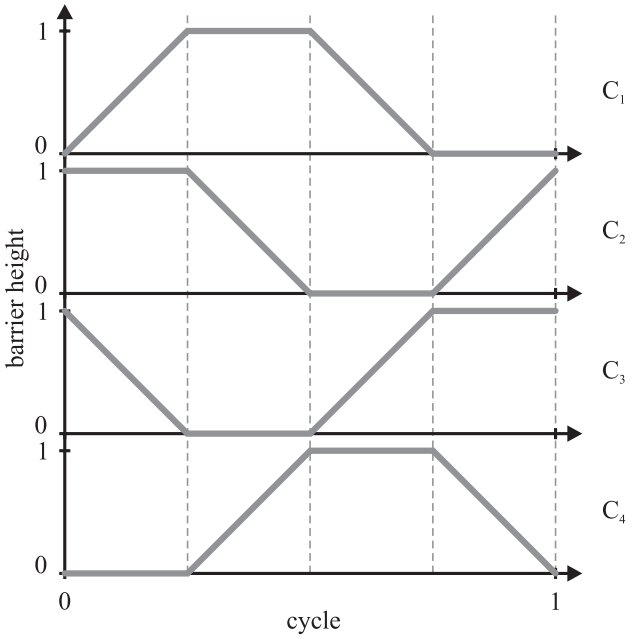

Fig. 4. The four possible phase shifted cyclic signals.

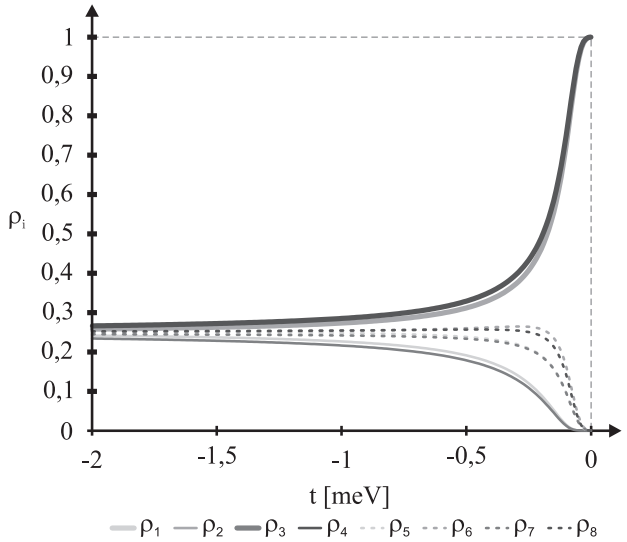

Fig. 5. The dependency of the single-electron density at individual sites, ρ_i, with respect to the tunnelling energy, for the case of a ternary quantum-dot cell transiting from neutral state to the state marked as A.

more arbitrary but typically -2 meV and corresponds to lowered barriers with a high probability of electron tunnelling between adjacent quantum dots.

When applied to the tQCA cells the cyclic signal presented in Fig. 3 turned out to be too abrupt. Indeed the signal changes the value of parameter t linearly, which compels a raising of the barriers that is too fast for proper localization of the electrons. This, as evident from Fig. 5, can to some degree be expected. In fact when raised barriers correspond to 0 meV and lowered to -2 meV one can notice that the switch phase of the cyclic signal is only a rough estimate of Fig. 5 and furthermore that most of the "action" happens when $t \in [-0.5, 0]$ meV. The increased number of quantum dots leads to more possible locations for the electrons to tunnel to, hence their localization in the desired

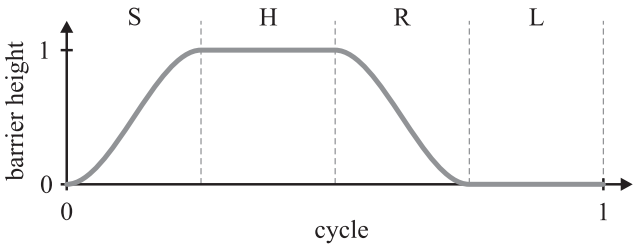

Fig. 6. The cyclic signal that controls the adiabatic switch in a ternary quantum-dot cellular automaton.

quantum dots is possible only when the barriers are sufficiently high.

We base the cyclic control signal for the tQCA cell on a sinusoidal function that was scaled to the interval [0, 1] (see Fig. 6). Dividing the function into two sections of equal length, one monotonically increasing the other decreasing we choose the first section as the control signal in the switch phase and the second as the control signal in the release phase. The hold and relax phase are kept unchanged (i.e., both are constant; the former with barriers completely raised and the latter with barriers absolutely lowered). The constructed signal has a more gradual change in the vicinity of raised barriers and thus allows more time for the electrons to successfully localize in the appropriate quantum dots.

4. Implementing the Ternary AND and OR Logic Functions

The initial approach that Lebar Bajec *et al.* took when developing ternary QCA structures was to use simple substitution of bQCA cells for tQCA cells.[8–10] With this assumption they hoped to obtain structures that will implement the same logic functions but in a ternary domain. They proved that using this simple approach the QCA wire and QCA inverter structures behave as intended, however, the structure that implements the binary logic functions AND and OR (i.e., the majority gate) proved to be more elusive.[8,11] Figure 7 presents the behaviour of the elusive structure, when following the abrupt switching principle and evaluating by means of the ICHA approach.

Ternary logic functions AND and OR can in the general multi-logic form be expressed as

$$\begin{aligned} y &= \mathrm{AND}(x_1, x_2) \equiv \min(x_1, x_2), \\ y &= \mathrm{OR}(x_1, x_2) \equiv \max(x_1, x_2), \end{aligned} \tag{7}$$

where x_1, x_2, $y \in \{0, 1/2, 1\}$.[16] Interpreting cell states A, B, and C as logic values 0, 1 and 1/2 respectively and comparing the behaviour of the structure presented in Fig. 7 with eq. (7) it can be noticed that with certain preconditions it is possible to extract a close representative of the desired truth tables. However, the preconditions that apply negate the so praised about property of the binary majority gate (i.e., that of using any arbitrary input as the selector of the gate's behaviour). Indeed in this structure it becomes evident that the only possible choice for the selector of the structure's behaviour is input S, whereas inputs X_1 and X_2 can serve only as inputs to the selected logic function. In other words, when input S is in state A (applied the logic value 0) and the states of inputs X_1, X_2, and output Y correspond with logic values of variables x_1, x_2 and y respectively, a truth table similar to that of a ternary AND logic function is obtained. The truth table similar to the one of the ternary OR logic function is, on the other hand, obtained when input S is in state B (applied the logic value 1).

A closer inspection of the structure's behaviour reveals also that state D cannot be assigned any logic value and that it seems not to serve any input-output function. Thus an additional, but easily met, precondition is that of state D being allowed only as an internal (processing) state.[8] Complying with these restrictions gives a truth table with only two erroneous input/output transformations where $\mathrm{OR}(1, 0) = \mathrm{AND}(0, 1) = \mathrm{D}$. Since the output is state D the authors suggest that an additional structure is to be constructed, which will convert this processing state to the correct output state. In a subsequent article they do precisely that.[10] They observe that the input/output transformations for $\mathrm{OR}(0, 1)$ and $\mathrm{AND}(1, 0)$, in other words, when the inputs are swapped, are correct and construct a hierarchy of three structures that produces the correct truth tables (see Fig. 8).[10] Although the presented structure successfully implements

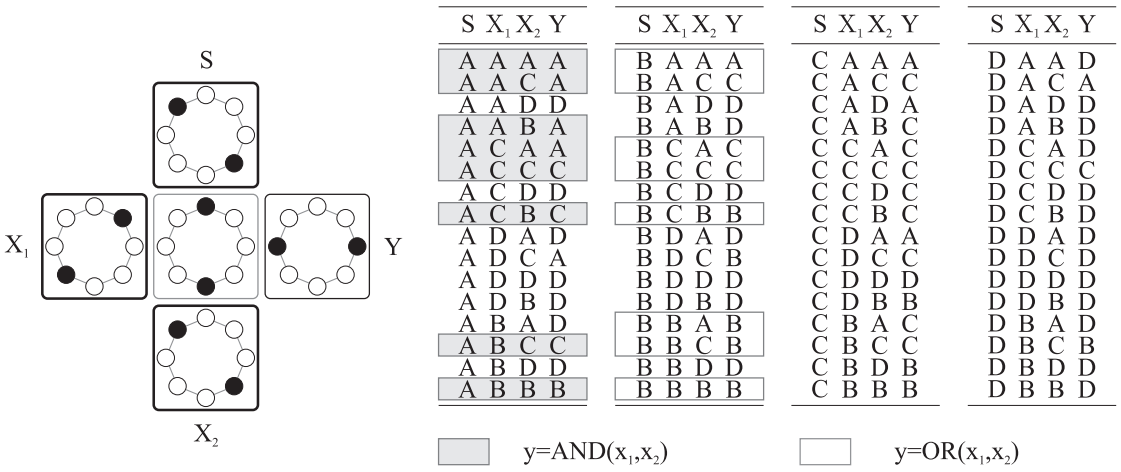

Fig. 7. The QCA based structure that was obtained by substituting the bQCA cells in the binary majority gate structure with tQCA cells (left). There are three input cells named S, X_1 in X_2, and one output cell named Y. The structure's truth table for all possible combinations of input cells' states (right).

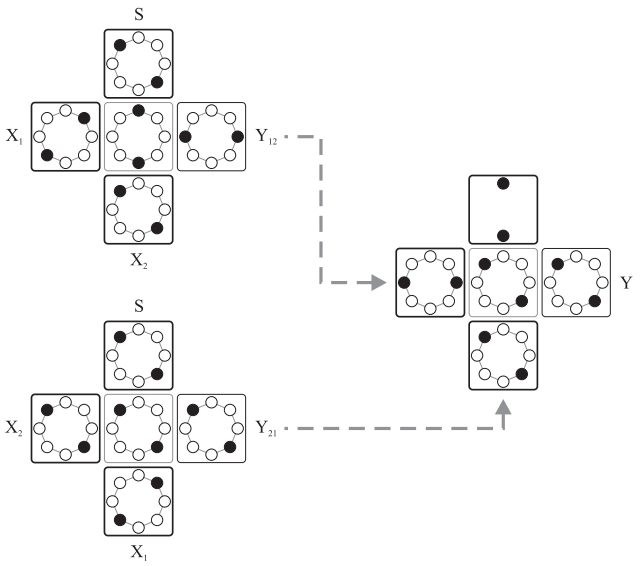

Fig. 8. The schematics of the structure proposed by Lebar Bajec *et al.* that implements the ternary AND and OR logic functions based on the principle of abrupt switching.

the ternary AND and OR logic functions it is, in view of the number of required cells, suboptimal. Indeed even when disregarding interconnections of the substructures the number of required cells tripled with respect to the binary counterpart. We believe that as the AND and OR logic functions are two of the most commonly used primitives when developing more complex structures the increased size could seriously hinder the success of ternary processing in QCA.

4.1 The ternary AND–OR gate

We here present a structure that is of the same size as its binary counterpart. What is more we present a structure that allows input flexibility (i.e., each of its inputs can be chosen as the selector for the structure's behaviour). We name it the ternary AND–OR gate.

The principal idea is to use the same geometry as for the binary majority gate, but promoted to pipelined processing based on adiabatic switching. Indeed a thorough analysis of the behaviour of the majority gate, when bQCA cells are substituted with tQCA cells, reveals that a possible source for invalid outputs are the cornering relations of the three inputs. The only two invalid output states are generated when the three inputs are symmetrical — A, B, A or B, A, B respectively. The idea is thus to first compute the maximum of the remaining two inputs when the third is in state A and minimum when it is in state B. After this initial step the intermediate result can safely be transferred to the output cell.

The above approach can be easily implemented using three phase shifted control signals C_1, C_2, and C_3 [see Fig. 9]. The barrier heights in the input cells obey signal C_1, the internal cell's signal C_2 and the output cell's signal C_3. When the inputs are in the hold phase the internal cell is in the switch phase (i.e., slowly transiting to a state that is in accordance with the states of all three inputs) and the output is in relaxed phase. When the internal cell is in the hold phase the output cell is in the switch phase whereas the inputs are in the release phase. This asserts that during the gradual raising of the barrier heights in the output cell the barrier heights in the input cells will be lowering and therefore at the instant of the output cell's highest "activity" the influence of the input cells will be minimal (in fact their states will be close to neutral). It is true that there is a delay of two phases between the application of the logic inputs to the input cells and the presence on the output logic value in the output cell, but as this is still in the same cycle we can observe the logic function as a time independent one. The truth table of the ternary AND–OR gate is presented in Table I. The restriction of state D being allowed only as an internal (processing) state is maintained. A close inspection of the first two columns reveals that the ternary AND–OR gate behaves as intended. If states A, B, and C are interpreted as logic values 0, 1, and 1/2 respectively and input cells X_1 and X_2 reflect the logic value of inputs x_1 and x_2 then the logic value of the output cell Y is $y = \min(x_1, x_2)$ when the state of input cell S is A (ternary AND) and $y = \max(x_1, x_2)$ when the state of input cell S is B (ternary OR). Analysing the truth table more thoroughly reveals also that the choice for the selector of the gate's behaviour is not limited solely to input cell S, but the ternary AND–OR gate

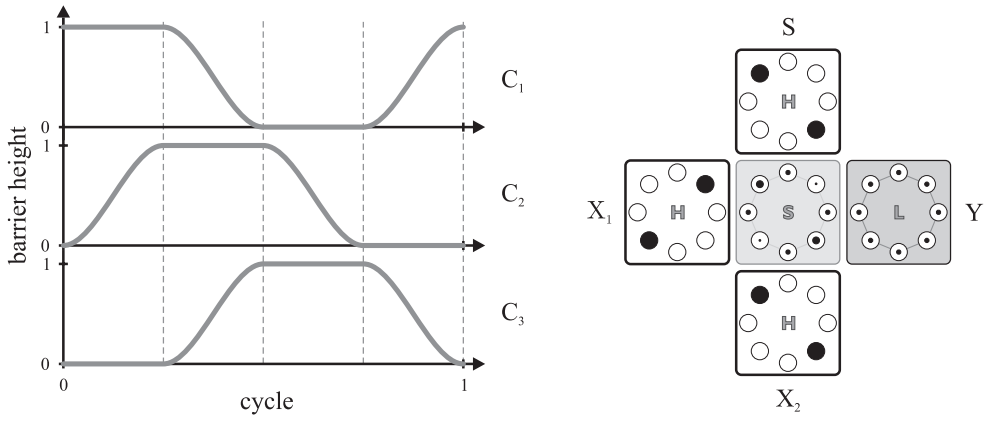

Fig. 9. The schematics of the structure that implements ternary AND and OR logic functions based on the principle of adiabatic switching. Cells marked with H (hold phase) are controlled by the cyclic signal C_1, those marked with S (switch phase) by C_2 and those marked L (relax phase) by C_3. A close inspection reveals that the size of the structure corresponds to the size of the structure, which implements the binary AND and OR logic functions.

Table I. The behaviour of the proposed structure when state D is allowed only as an internal (processing) state.

S	X_1	X_2	Y	S	X_1	X_2	Y	S	X_1	X_2	S
A	A	A	A	B	A	A	A	C	A	A	A
A	A	C	A	B	A	C	C	C	A	C	C
A	A	B	A	B	A	B	B	C	A	B	C
A	C	A	A	B	C	A	C	C	C	A	C
A	C	C	C	B	C	C	C	C	C	C	C
A	C	B	C	B	C	B	B	C	C	B	C
A	B	A	A	B	B	A	B	C	B	A	C
A	B	C	C	B	B	C	B	C	B	C	C
A	B	B	B	B	B	B	B	C	B	B	B

computes the ternary conjunction between the remaining two inputs whenever the third is in state A and ternary disjunction whenever it is in state B.

5. Conclusions

In this article we present a quantum-mechanic description of the ternary quantum-dot cellular automaton. We base it on an extended Hubbard-type Hamiltonian. The model allowed the introduction of the adiabatic switching principle. The intercellular Hartree approximation is employed to study the influence of the barrier heights on the localisation of electrons in a ternary quantum-dot cell. The knowledge gained was applied to the design of the cyclic control signal used in adiabatic switching. The latter allowed the development of the ternary AND–OR gate; a ternary quantum-dot cellular automaton that can compute the ternary AND and OR logic functions. The advantage of the developed structure over the existing ones is that it, with respect to the number of required cells, equals the binary majority gate. A further advantage is that it allows using one of its three inputs as selector of the gates behaviour. Whenever one input is applied the logic value 0 the gate computes the conjunction of the remaining two inputs and whenever it is applied the logic value 1 the gate computes the disjunction of the remaining two inputs. The structure can be used as a primitive when constructing both binary or ternary complex structures which opens up the possibility to design advanced ternary arithmetic-logic and memorizing units, the building blocks of ternary processors. It has to be noted that in this work we assume that there are no technological limitations and neglect practical time scales of adiabatic switching principally in favour of focusing attention on processing architectures. Nevertheless, we are well aware of implementation problems and for this reason the switching dynamics and material suitability are part of our current research directive.

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