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Towards the bottom-up concept: extended quantum-dot cellular automata

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Abstract

In this article we present an extended *quantum-dot cellular automaton* (QCA) cell. The classical QCA cell is extended in the sense of an enlarged range of its possible stable and usable states. Indeed, in the classical QCA cell the electrons, owing to electrostatic repulsion, align along one of the two diagonal configurations that correspond to their maximal spatial separation. The QCA cell thus has the ability to encode two states - two logic values (0 and 1). By extending the QCA cell with four additional quantum dots we introduce the extended QCA (EQCA) cell and analyze its behavior, the analysis of which is based on the semi-classical modeling approach. Experiments showed that by using a special interpretation of electron configurations in the EQCA, the range of possible states can be increased from two to three, giving the EQCA cell the ability to encode the logic values (0, ½ and 1). The primary motive of this article is to promote the idea of finally switching focus from pure miniaturization and the top-down concept to the bottom-up concept and start extending the currently available approaches to allow for "richer" processing and data storage capabilities without a major increase in space requirements.

Keywords: quantum-dot cellular automata; multi-valued logic; switching structures;

1. Introduction

According to the well-known prediction made by Gordon Moore forty years ago, the increase in the number of transistors per square inch of integrated circuits doubles every 18 months. With this pace of miniaturization it is to be expected that in the next five to ten years the integration will be at the nanometer scale [1]. Due to this fact, many researchers have focused on this problem. Indeed, in

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the early 1990s Lent et al. [2] demonstrated a possible interpretation of configurations of a pair of tunneled electrons contained in a quantum-dot cell as the logic values 0 and 1. Later studies of the behavior of spatial arrangements of quantum-dot cells denoted as *quantum-dot cellular automata* (QCA), resulted in the implementation of the binary wire and the functionally complete set of logic functions [3]. The ability to transfer data and the functionally complete set of logic functions enables the construction of any given switching structure. However, the focus of QCA research was, and still is, mostly dedicated to the implementation of the binary logic and the corresponding computer structures associated with it. This results from the fact that the basic building blocks (i.e. the QCA cells) are still capable of representing only 1 bit of data (i.e. either the logic value 0 or 1). In this article the semi-classical modelling approach [4] is employed to study an eight-dot QCA cell, denoted as the extended QCA (EQCA) cell. It has to be noted, however, that this article is mostly theoretical. Its primary goal is the promotion of the idea to switch focus from top-down concepts (e.g. pure miniaturisation) to bottom-up ones (e.g. search for ways that enable "richer" processing and data storage capabilities by employing currently available means). Our first attempt in this direction is the study case of the EQCA cell, which, as shown, can be used to represent three logic values: 0, ½ and 1.

The article starts with a condensed overview of the classical QCA cell and the corresponding two valued switching structures. It then continues in section 3 with the introduction of the EQCA cell and the study of EQCA based switching as a possible approach to ternary logic.

2. The quantum-dot cell

The classical QCA cell has the structure presented in Fig. 1a with four quantum dots separated by tunneling barriers. Due to electrostatic repulsion, the electrons tend to align along one of the two configurations corresponding to their maximum spatial separation. If no external electric field is present, the configurations have exactly the same energy. In the presence of a nearby cell with a well-

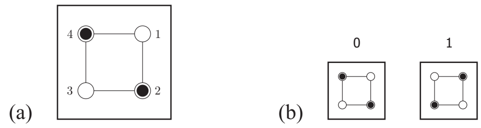

Fig. 1: Layout of the four-dot QCA cell with two electrons (a) and the two distinct electron configurations corresponding to their maximum spatial separation (b).

defined charge distribution, however, one of them is energetically favored. Traditionally the two configurations are associated with logic values 0 and 1 as presented in Fig. 1b, while the rest of all possible configurations are not associated with any logic value.

2.1. QCA based switching

A *QCA structure* is a spatial arrangement of QCA cells that can be decomposed into three segments. The first segment represents the input cells or *drivers*.[1] In a physical sense these are usually located at the extremes of the structure and their states are enforced by means of external electric field sources. The second segment is represented by the *internal* cells. These, with respect to their physical arrangement within the structure, transform and transmit data from the input cells towards the third segment; the output or *target* cells. A more detailed description of the basics of QCA structures together with the temporal dynamics and synchronization mechanisms can be found in [2,5-7].

When several cells are lined up to form a wire, and a given logic value enforced to the input cell, it will propagate along the wire in a domino fashion [3], until all cells, including the target cell, have reached the same configuration (Fig. 2). Since the

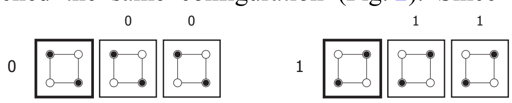

Fig. 2: The binary-wire; propagation of enforced logical values 0 and 1 along a line of QCA cells.

former means that the logic value 0 or 1 enforced (input) on one end of the wire is propagated to its other end (output), the wire is called a *binary wire*.

[1] In Figs. 2-4 and 6 a driver cell is represented with a bold outline.

The motivation for designing QCA structures originates, however, from the aspiration to find arrangements of cells that implement the functionally complete set of logic functions AND, OR and NOT [8]. In this Lent et al. [2] succeeded first. The QCA inverter, or NOT, was obtained by offsetting the target cell from the driver cell by 45° as shown in Fig. 3. In this case a given logic value enforced to the

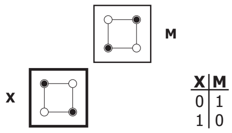

Fig. 3: The QCA inverter.

driver appears inverted in the target cell, the logic value 0 thus producing 1 and vice versa. The AND and OR function, on the other hand, were constructed as an intersection of three binary wires [2,9]. This produces the topology presented in Fig. 4, known as

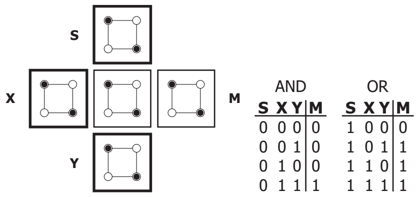

Fig. 4: The QCA majority gate.

the *QCA majority gate*. In this gate the three drivers (S, X and Y) 'vote' on the configuration of the internal cell and the majority wins. The configuration of the internal cell is then propagated toward the target cell (M). The majority gate is special, because one of the drivers (in our case S) can be used to select the gates' behavior. Indeed, if this driver is enforced with the logic value 0, then the gate behaves as the AND logic function, whereas if it is enforced with the logic value 1, the gate behaves as the OR logic function.

The ability to transfer data and the functionally complete set of logic functions gives us the ability to construct any given switching structure and thus enables QCA computation.

3. The extended quantum-dot cell

The EQCA cell is an eight-dot QCA cell with the structure presented in Fig. 5a. The dots are evenly

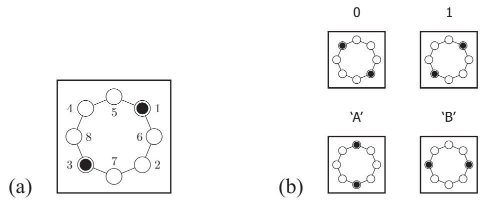

Fig. 5: Layout of the eight-dot EQCA cell with two electrons (a) and the four distinct electron configurations corresponding to their maximum spatial separation (b).

distributed in a circular fashion, and again separated by tunneling barriers. Similar to the QCA cell the electrons will, due to the electrostatic repulsion, tend to align along one of the configurations that correspond to their maximal spatial separation.[2] There are four distinct configurations with maximal spatial separation between electrons. For the diagonal configurations we shall use the same association as in the classical QCA case. The configuration with electrons in dots 2 and 4 is thus associated with the logic value 0 and the configuration with electrons in dots 1 and 3 with the logic value 1. The vertical configuration (i.e. electrons in dots 5 and 7) and the horizontal configuration (i.e. electrons in dots 6 and 8) are, on the other hand, denoted as configurations 'A' and 'B' as presented in Fig. 5b.

3.1. EQCA based switching

Like in the case of the binary-wire, when several cells are lined up to form a wire and either the logic value 0 or 1 is enforced to the driver cell, it will propagate along the wire, until all cells have reached the same configuration. Modeling after [4] showed that 'A' or 'B' will, however, propagate along the wire in an alternating fashion, such as for example ABABABAB… (see Fig. 6). By associating both

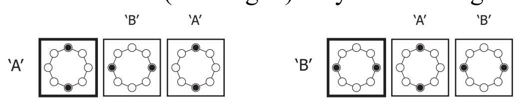

Fig. 6: The ternary-wire; propagation of enforced configuration 'A' or 'B' along a line of EQCA cells.

[2] Have in mind that our primary interest is to generalize the functionality of a cell in the sense of an expanded range of possible states. Currently, however, both the process of physical construction and the process of forcing/detecting a charge distribution are taken out of account.

configurations ‘A’ and ‘B’ with the logic value ½ the functionality of the wire gets extended, making it capable of propagating three logic values 0, ½ and 1; making it a *ternary-wire*.

All that is needed now is a functionally complete set of logic functions that enables ternary logic. Our approach was to test if the QCA structures, used to implement the binary logic functions, would give the correct results even when constructed by using EQCA cells. In other words, to test if they behave in accordance with the truth tables set up by Jan Łukasiewicz [10], or more generally in accordance with the equations: NOT(x)=1-x, AND(x,y)=min(x,y), OR(x,y)=max(x,y).

Surprisingly enough, the inverter, or NOT, worked perfectly, giving the correct output for any given input. In the case of input logic values 0 and 1 it behaved just as the QCA inverter. Then again, if the configuration enforced to the driver was either ‘A’ or ‘B’, the configuration of the target cell did not change at all, which is precisely what was expected.

The EQCA majority gate, on the other hand, proves to be more elusive. Indeed the range of possible combinations of driver configurations increases from 2^3=8, in the QCA case, to 4^3=64 in the EQCA case.[3] Looking at the complete list, it becomes evident that the majority gate does not work as intended. However, marking ‘B’ as a processing configuration (i.e. not allowing it for driver/target cells, but only for internal cells)[4] gives the truth table presented in Fig. 7. This truth table is remarkably similar to the one set up by Łukasiewicz. The only problematic input/output combinations are AND(1,0) and OR(0,1), which both reach configuration ‘B’ instead of returning 0 and 1 respectively. Even so, the two cases are remarkably symmetrical. Since configuration ‘B’ is marked as a processing configuration, a possible solution for obtaining the required truth table would be to append an additional EQCA structure. Its sole objective would be to translate configuration ‘B’ into the correct output value. Our current research is focused upon designing such a structure.

AND

S	X	Y	M
0	0	0	0
0	0	½	0
0	0	1	0
0	½	0	0
0	½	½	½
0	½	1	½
0	1	0	‘B’
0	1	½	½
0	1	1	1

OR

S	X	Y	M
1	0	0	0
1	0	½	½
1	0	1	‘B’
1	½	0	½
1	½	½	½
1	½	1	1
1	1	0	1
1	1	½	1
1	1	1	1

Fig. 7: The EQCA majority gate truth table.

[3] With the given interpretation of configurations ‘A’ and ‘B’ (i.e. both as the logic value ½) the number of distinct input combinations is in fact 3^3=27.

[4] It has to be noted, that this precondition is easily met. Indeed, whenever a logic value is required to be transmitted over a EQCA wire, the wire must be constructed from an odd number of EQCA cells, then the target cell will always assume the same configuration as the driver cell, even in the case when the configuration enforced to the driver is ‘A’ or ‘B’.

4. Conclusion

In this article we present an extended quantum dot cellular automaton cell. The extension is focused on an enriched range of its possible states. It is shown that this cell is capable of representing three logic values 0, ½ and 1, as well as that it is capable of propagating them along a line of cells. In addition it is shown that by marking one of the configurations as a processing configuration results in only two erroneous input/output combinations of the ternary AND and OR logic functions. Our current research is focused on solving this problem.

References

[1] A.M. Steane, E.G. Rieffel, IEEE Computer 33 (2000) 38.

[2] C.S. Lent, P.D. Tougaw, W. Porod, G.H. Bernstein, Nanotechnology 4 (1993) 49.

[3] P.D. Tougaw, C.S. Lent, J. Appl. Phys. 75 (1994) 1818.

[4] M. Macucci, G. Iannaccone, S. Francaviglia, B. Pellegrini, Int. J. Circ. Theor. Appl. 29 (2001) 37.

[5] C.S. Lent, P.D. Tougaw, J. Appl. Phys 74 (1993) 6227.

[6] M.T. Niemier, P.M. Kogge, in: S.K. Shukla and R.I. Bahar (Ed.), Nano, Quantum and Molecular Computing, Kluwer Academic Publishers, Boston, 2004 (Chapter 9).

[7] P.D. Tougaw, C.S. Lent, J. Appl. Phys. 80 (1996) 4722.

[8] Z. Kohavi, Switching and finite automata theory, McGraw-Hill Inc., New York, 1978.

[9] G.L. Snider, A.O. Orlov, I. Amlani, G.H. Bernstein, Jpn. J. Appl. Phys. 38 (1999) 7227.

[10] L. Borkowski, Jan Lukasiewicz: Selected Works, North-Holland Publishing Company, Amsterdam, 1970.