
ORGANIZED FLIGHT IN BIRDS

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ABSTRACT

1 The organized flight of birds is one of the most easily observed, yet challenging to study,
2 phenomena in biology. Birds that fly in organized groups generally do so in one of two fashions:
3 Line formations and Cluster formations. The former groups are typically demonstrated by large
4 birds like waterfowl, where birds fly arranged in single lines, often joined together. The
5 scientific questions about these groups usually involve potential adaptive functions, such as why
6 do geese fly in a V? The latter, Cluster formations, are typically made up of large numbers of
7 smaller birds like pigeons or blackbirds flying in more irregular arrangements that have a
8 strong three dimensional character. The groups are defined by synchronized and apparently
9 simultaneous rapid changes in direction. Scientific questions about these groups are usually
10 concerned with mechanism; how is synchrony achieved? Although field observations about the
11 phenomenon date to the origins of natural history, experimental studies did not begin until the
12 1970s. Early experimenters and theoreticians were primarily biologists, but more recently
13 aeronautical engineers, mathematicians, computer scientists, and currently, physicists have
14 been attracted to the study of organized flight. Computer modelling of organized flight has
15 recently generated striking visual representations of organized flight and a number of
16 hypotheses about the functions and mechanisms of organized flight, but the ability to test these
17 hypotheses lags behind the capacity to generate them. It is suggested that a multiple-
18 disciplinary approach to the phenomenon will be necessary to resolve apparently conflicting
19 current hypotheses.

20 Keywords

21 animat; bird aerodynamics; bird flight; bird flocking; boid; cluster formations; Canada Goose;
22 *Branta Canadensis*; European Starling; *Sternus vulgaris*; flight formations; flock simulations;
23 flocking simulations; line formations; V formation

24 The orderly aerial manoeuvres of birds have fascinated and mystified observers since the
25 beginnings of written natural history 2,000 years ago, when Pliny suggested that geese ‘...travel
26 in a pointed formation like fast galleys, so cleaving the air more easily than if they drove at it
27 with a straight front’ (Rackham 1933). Why do geese fly in a V, and how do pigeons all seem to
28 be able to take off and turn at once? The study of these phenomena offers an encapsulated
29 model of the development of knowledge of other behaviours, starting with anecdotal
30 descriptions and speculation, measured observations of increasing precision, formation of
31 testable hypotheses, and then tests of these hypotheses. In the case of the study of organized
32 flight in birds, the first phase began at about the beginning of the twentieth century, the second
33 and third in the 1970s and the fourth in the mid 1980s. The study of bird organized flight also
34 offers a good demonstration of Kuhn’s (1962) suggestion that science advances in saltatory
35 fashion, each ‘revolution’ being prompted by a new technique or apparatus that allows old data
36 to be looked at in a new way.

37 The early investigators of organized flight were, with a few notable exceptions, biologists. In
38 the 1970s, aeronautical engineers started to be attracted to the phenomenon, followed by
39 computer scientists in the 1980s, and physicists and mathematicians in the 1990s. These later
40 investigators have been primarily interested in modelling the behaviour. The fraction of active
41 investigators with a biological background has steadily decreased over the years. We will try to
42 demonstrate that as the elegance of models has increased, so has their distance from behaviour
43 in the field, and that future progress in the area will depend on collaborations between
44 physicists, mathematicians, computer scientists and biologists rather than specialists working
45 alone.

46 **THE ERA OF ANECDOTE AND SPECULATION**

47 Several ornithologists of the 1930s made visual field observations that would later be very
48 provocative to experimentalists and theoreticians. Nichols (1931) noted that in turning and
49 wheeling pigeon, *Columba livia*, flocks, the position of the birds at the head of a turning flock

50 would be exchanged with birds at the side after the completion of a turn; there did not appear to
51 be consistent 'leadership' in such flocks. He speculated that this behaviour might be the result of
52 faster birds in the front of the formation moving ahead of the flock, then turning back to rejoin.
53 The visual stimulus provided by the turnaround might provide a signal for the rest of the birds
54 to turn, apparently simultaneously. He suggested that a change in direction was related to a
55 change of positional leadership.

56 Selous (1931) made a 30-year series of meticulous visual observations on various species of
57 birds flying in organized flocks, and was convinced that within the limits of unassisted human
58 vision, there were occasions when birds rose from the ground, or made turns simultaneously.
59 He concluded that there could be only two possible explanations for such a phenomenon;
60 disturbance from outside the flock, say the sight of a predator, which would be instantaneously
61 received by all birds in the flock, and would be reacted to in identical manner, or an undefined
62 quality he called 'thought transference', or what we might call today 'telepathy'.

63 Selous appeared convinced that there were at least some occasions when groups of birds
64 would rise from the ground, apparently spontaneously, with no discernible source of outside
65 disturbance. He also noted in contrast that there were times when a flock on the ground would
66 be indifferent to the rapid approach of an aerial predator, as when members of a flotilla of
67 Eurasian Coots, *Fulica atra*, leisurely swam away as a Great Black-backed Gull, *Larus marinus*,
68 made a low pass over their group. Penrose (1949) made a similar observation when he dove
69 from above toward a large European Starling, *Sternus vulgaris*, flock in a sailplane.

70 Selous also noted that flocks on the ground would sometimes take to the air in a stepwise
71 fashion. Individuals or small groups of Black-headed Gulls, *Larus ridibundus*, would take flight
72 without any discernible effect on neighbours, and then with no obvious temporal relationship to
73 previous small group departures, the entire remainder of the flock, hundreds of birds, would
74 take flight simultaneously.

75 'Thought transference' had a different standing in the scientific community in Selous' time
76 than it does today, and it is not surprising that, for want of a better explanation, a careful

77 observer like Selous might be led to something as heterodox as telepathy to explain an
78 otherwise inexplicable phenomenon. Rhine (1983) had started reporting the results of
79 parapsychology experiments using conventional experimental design in 1927, and England,
80 where Selous made his observations was a centre of interest in 'paranormal' phenomena. Selous
81 never explored what the nature of thought transference might be.

82 Gerard (1943) was one of the first individuals to try to quantify turning behaviour in a flock.
83 Whilst pacing a group of approximately 100 unidentified birds in a car being driven at 35 mph
84 (60 km/h), he observed that the entire flock turned left in a flanking movement, rather than a
85 column movement, in military parlance. In a flanking movement all individuals turn at once
86 upon the signal to do so, rather than advancing to a defined point and then turning. He
87 speculated that no bird advanced more than a body's length beyond any other bird before
88 turning, by his calculation within 5 ms of any other bird. Assuming a minimum reaction time of
89 100 ms, he proposed that any coordinating signal must have been acted on with great constancy
90 by receiving individuals. Gerard's own vision must have been remarkable to be able to make
91 this observation while driving a car, but his estimate of probable reaction time was very close to
92 Pomeroy & Heppner's (1977) laboratory study results of startle reaction times in the European
93 Starling of 70 ms.

94 Much of the early work on flight flocking was devoted to considerations of the biological
95 utility of flocking, from an ecological or behavioural standpoint, rather than the perspective of
96 organizing principles or mechanisms. Beer (1958) questioned whether large groupings of birds
97 had 'any' distinctive utility, and were merely 'haphazard organizations'. Vine (1971), on the
98 other hand, suggested that a circular grouping provided the best predator avoidance strategy
99 against visual predators. Emlen (1952) looked at flocks from the ethological perspective of the
100 times, and suggested that both flocking itself, and the structure of the flock resulted from the
101 interplay of attractive and repulsive behavioural forces.

102 One of the annoyances that has persisted over the years for those studying flocks is an
103 etymological one; there has been no consistency in the literature in terms of the definition of

104 'flock' and categories of same. The difficulty is not a trivial one. One author might be describing
105 the properties of a class of behaviours that is quite different than those studied by a different
106 investigator, but both will use the same term.

107 For example, Emlen (1952, p. 160) described a flock as 'any aggregation of homogeneous
108 individuals, regardless of size or density'. This definition immediately presents difficulties,
109 because there are very common aerial groupings, such as mixed blackbird groups, composed of
110 different species. Beer's (1958, p. 78) definition of a flock was '...two or more birds which
111 associate with each other due to innate gregarious tendencies'. This definition breaks down in
112 the face of more recent flocking studies, like Reynolds' (1987), which suggest that coordinated
113 flocking may be the product not simply of 'gregariousness', but extremely simple behavioural
114 rules followed by each bird in the group.

115 Heppner (1974) developed a taxonomy of airborne bird flocks. The primary dichotomy in this
116 scheme was between 'Flight Aggregations', which are unorganized groups of flying birds
117 gathered in an area for a common purpose, such as gulls circling about a fishing trawler, and
118 'Flight Flocks', which were organized groups of flying birds coordinated in one or more aspects
119 of flight, such as taking off, turning, landing, etc. However, these distinctions seem not to have
120 been universally adopted in the literature; one regularly sees the term 'aggregation' used to
121 describe what Heppner would have called a 'flight flock'.

122 Heppner's second order division of 'Flight Flocks' has demonstrated some persistence and
123 consistency in the literature. He differentiated flight flocks into 'Line Formations' and 'Cluster
124 Formations' (Fig. 1). Line formations are demonstrated by relatively large birds that fly in
125 regular lines or queues, such as geese, cormorants, or ducks. Cluster formations have a three
126 dimensional structure like a sphere, and are typically seen in smaller birds like pigeons,
127 starlings, and smaller shorebirds. Interestingly, line flying birds like geese may sometimes be
128 seen in a cluster, but cluster flying birds like starlings are rarely, if ever, seen flying in single
129 lines.

130 The categories of biological questions that are raised by each of these formations are quite
131 different. Typically, 'how' questions are raised about cluster flocks. Do the birds really turn all at
132 once? How can they achieve synchrony in taking off and landing? How do they decide when to
133 turn, and in what direction? 'Why' questions are more characteristic of line flying birds. What
134 might be the biological advantage of flying in this configuration? Are there energy savings to be
135 had? Does the formation shape facilitate communication? A broad question that might apply to
136 both groups is whether there is a general advantage to flying in groups, as opposed to solitary
137 flight?

138 A literature search suggests that investigators recognize that the two formation categories
139 may represent quite different biological issues. Early key papers on line formations tend to be
140 cited through generations of papers on line formations, but not cluster formation studies, and
141 vice versa. For this review, we recognize the difference between these lines of investigation, and
142 will treat them separately.

143 **LINE FORMATIONS**

144 Line flying birds typically fly in staggered, or 'echelon', formations rather than in straight lines
145 nose-to-tail. If two such formations are joined at an apex at the front of the formation, we have a
146 V or a J, its asymmetric variant. Franzisket (1951), von Holst (1952) and Hochbaum (1955)
147 suggested that close formation flight might provide the advantage of a turbulence free zone
148 behind a bird ahead, but that would seem to apply only if the birds flew immediately behind the
149 bird in front, like race cars, which they rarely do.

150 Two competing, but not necessarily mutually exclusive, hypotheses have been advanced to
151 explain the functionality of staggered line formations (most of the papers to be subsequently
152 cited here refer to V formations and their properties, but Gould & Heppner [1974] found in a
153 study of 104 Canada Goose, *Branta canadensis*, formations that Vs and Js together were less
154 common than single staggered lines, or echelons. Additionally, O'Malley & Evans [1982a, b]
155 found that White Pelicans, *Pelecanus erythrorhynchos*, flying in line formations only flew in Vs

156 10% of the time). Wieselsberger (1914), an aerodynamacist, was the first to suggest an
157 aerodynamic advantage for line formation flight. He proposed that a V partitioned drag equally
158 between the two legs of the flight, and that birds flying to the left or right of a bird in front could
159 recapture energy lost to an 'upwash' generated off the wingtips of the preceding bird. The
160 alternate, behavioural view suggests that social and perceptual factors have guided adoption of
161 staggered formations. Hamilton (1967) suggested that flying in staggered lines permitted the
162 optimum combination of visibility of neighbours, and a clear visual field to the front. Heppner
163 (1974) suggested that the fixed position of the eyes in the heads of line-flying birds might make
164 a staggered formation advantageous for keeping the image of an adjacent bird on the visual axis
165 of a given bird's eyes. However, Heppner et al. (1985) found that the angle between the legs of a
166 V formation of Canada Geese that would place the image of a leading bird on the visual axis of
167 the eyes of a following bird (128°) was considerably more obtuse than the measured V-angles of
168 V-formation flying birds in previous studies (Gould & Heppner 1974; Williams et al. 1976;
169 O'Malley & Evans 1982a). They also noted that although Canada Geese have a limited amount of
170 binocular vision to the front, despite having eyes located on the sides of their heads, they also
171 have a 'blind cone' in back of them of 29° on either side of the midline. Thus, a V angle of 58° or
172 greater would enable every bird in the formation to see every other bird, even those following
173 behind.

174 Warnke (1984) offered a third hypothesis that, judging by the number of subsequent
175 citations, seems not to have generated much enthusiasm in the V formation community. He
176 suggested that the V formation could be explained by the interaction of electrostatic fields
177 generated by flapping flight. He did not discuss how birds would be able to detect such fields,
178 nor did he explore the advantage that might accrue to a bird by basing its proximity to a
179 neighbour on the basis of these fields. There was much interest in the biological effects of
180 electromagnetic fields in the 1970s and 1980s; Heppner & Haffner (1974) suggested that
181 coordinated cluster flocks might be explained by signals sent by a leader to all birds in a flock by
182 means of a hypothetical radiated electromagnetic field. Interestingly, Hill (1972) described a

183 device he patented that was a wing-levelling autopilot for model aircraft that operated by the
184 differential in electrostatic fields between wingtips of a moving model airplane, so Warnke's
185 idea was not totally implausible.

186 There have been more papers addressing the aerodynamic hypothesis of staggered flight than
187 other hypotheses, and a bit of aerodynamic theory here will make the subsequent references
188 more intelligible. For a bird to fly by use of a wing requires a 'relative wind'; a passage of air
189 over the wing. If the front of the wing is tilted up slightly relative to the wind, the relative wind
190 is deflected downward. The result is a positive force on the underside of the wing, 'Newtonian
191 lift' (Fig. 2). Additionally, airplane wings are typically constructed so that the top of the wing is
192 curved and air moves faster over the top of the wing, creating a negative pressure on the top of
193 the wing; 'Bernoulli lift'. In still air, we must generate the relative wind by moving the aircraft
194 forward. To do so, we must have a force called 'thrust', generated, for example, by a propellor. In
195 general, the faster the aircraft goes, the more lift is generated by the wing. Unfortunately, as the
196 aircraft accelerates, 'drag' is produced, at least in part by friction between the air and the
197 surface. One type of drag, 'induced drag', is especially germane to bird flight. Lift is, partly,
198 created by the angle with which the wing meets the air (referred to also as the angle of attack).
199 Up to a limit, the steeper the angle, the greater the lift—but also, the greater is the induced drag,
200 which is produced as a by-product of lift. Compared to airplanes, birds are typically low-speed
201 aircraft whose wings produce a lot of induced drag.

202 The inner part of a bird's wing provides most of the lift, the outer part, by a kind of rowing
203 action, provides the thrust. As air streams over a wing generating lift, it tends to form vortices,
204 which typically stream off the wing as 'tip vortices', essentially horizontal tornadoes. These tip
205 vortices have a rising and falling component, and in an airplane, may carry sufficient energy to
206 upset a smaller aircraft following a larger one in for a landing. It is this energy, which essentially
207 represents a cost of flight using wings, that the aerodynamic hypothesis of V formation suggests
208 might be partially recaptured by a following bird whose own wingtip was located in the upward
209 rising part of the tip vortex, or upwash, streaming off the wing of the preceding bird (Fig. 3). The

210 diameter of the vortex increases with distance from the producing bird's wingtip, and tends to
211 dissipate with increasing distance. The placement of a following bird's wingtip in relationship to
212 the vortex from a preceding bird's wingtip should, in theory, affect how much energy is
213 recaptured by the following bird. To recapture tip vortex energy from a preceding bird, a
214 following bird would have to be positioned to the left or right of a preceding bird, suggesting
215 that a V (or at least a staggered, or echelon) formation would be advantageous for birds flying in
216 a group.

217 Lissaman & Schollenberger (1970) produced the first quantitative suggestion, based on
218 aerodynamic theory, of exactly how much energy might be saved by a group of birds flying in a
219 V formation. They proposed that a group of 25 (unspecified species) birds flying in a V would
220 have 71 percent more range than a single bird. Their optimum V angle appeared to be about
221 120° between the legs of the V. For later investigators, this paper was both stimulating and
222 frustrating because they did not present the calculations and formulae used to arrive at their
223 conclusions, ignored the quantitative effects of flapping rather than fixed wing flight, and did not
224 apparently consider the difference between air flowing over a smooth metal surface and a
225 feathered wing nor the aerodynamic scaling effects of small birds flying at low speeds compared
226 to aircraft. Nonetheless, this paper provided a 'target' for experimental and quantitative
227 observational work.

228 Haffner (1977) flew Budgerigars, *Melopsittacus undulatus*, in a wind tunnel and used a smoke
229 stream to visualize the airflow over the bird's wing. He concluded that flapping wing flight is
230 aerodynamically complex, and that calculations of energy saving for the V formation using fixed
231 wing models were oversimplified, and probably overgenerous. Using Cone's (1968) theoretical
232 studies on flapping wing flight and his own experimental work, he concluded that potential
233 energy saving of V formation flight compared to solitary flight was a much smaller maximum of
234 22%.

235 Willis et al. (2007) examined the theoretical energy savings in formation flight with respect to
236 basic positioning and wing beat phase relationships between a preceding bird and a following

237 bird. Nachtigall (1970) found a phase synchrony in a field study of wing beats in Canada Goose
238 formations, but Gould (1972), in a similar study, failed to do so. Willis et al.'s (2007) study is
239 preliminary as they do not consider the optimal formation shape or detailed flapping kinematics
240 or wing shapes. Nonetheless, their results suggest that optimal flapping phase synchrony
241 accounts for up to 20% of induced flight power savings, but that precision phase locking is not
242 required for energy savings to occur. They also observed that ideally, the following bird would
243 not be vertically elevated above or below the lead bird's wake if flapping started in phase. If
244 flapping is not in phase, however, it may be advantageous to take on a vertical displacement
245 relative to the preceding bird to most effectively capture its strongest upwash regions. They
246 suggest that vertical displacements in nature probably do not happen for aerodynamic benefit,
247 as for that to occur precision flight dynamics and sensing would be required.

248 Determination of the distance between birds, and the angle of the legs of the V would be
249 necessary to test V formation hypotheses. Gould & Heppner (1974) performed the first field
250 measurement of both parameters in Canada Geese using projective geometry and still
251 photography. They reported a mean angle between the legs of the V \pm SD of $34\pm 6^\circ$, $N=5$, with a
252 mean distance between bird bodies \pm SD of 4.1 ± 0.8 m, $N=3$ and a mean flock size \pm SD of 18 ± 12
253 birds, $N=5$. Two years later, Williams et al. (1976) examined V angles in Canada Goose
254 formations using a radar technique. They found a range of $38\text{--}124^\circ$ in the feeding flights they
255 recorded. Further, they noted that the angle in a single formation varied from $5\text{--}40^\circ$ between
256 successive sweeps of the radar beam (duration of sweep not reported). Both groups of authors
257 used their respective photographic and radar techniques on the same flocks of birds in 1975,
258 and found no significant difference between the two.

259 Higdon & Corrsin (1978) refined Lissaman & Schollenberger's (1970) hypothesis by
260 considering the effects of flying in three-dimensional fashion, i.e. in a cluster, like starlings. As
261 one might suspect, the physics is considerably more complex, but they suggested that it was
262 aerodynamically disadvantageous to fly directly behind another bird, and that a tall, narrow
263 cluster flock (such as is often seen in mixed blackbird flocks) is aerodynamically

264 disadvantageous compared to solitary flight. May (1979) also re-examined Lissaman &
265 Schollenberger's (1970) suggestions, and concluded that the aerodynamic advantage of line
266 flight in large birds was 'slight', perhaps as little as 10% compared to solitary flight.

267 Badgerow & Hainsworth (1981) re-examined Gould & Heppner's (1974) data on distances
268 between Canada Geese to obtain 'wingtip spacing', a variable they felt was more appropriate
269 than 'distance between body centres' in testing the aerodynamic hypothesis of V formation
270 flight. When they did this, they found a number of birds had wingtips that overlapped the
271 position of the wingtips of a bird ahead, a problematic situation for producing an energy
272 advantage in Lissaman & Schollenberger's (1970) hypothesis. In contrast to Lissaman &
273 Schollenberger's (1970) predicted maximum range increase of 71% for V formation flight,
274 Badgerow & Hainsworth's (1981) revision predicted a maximum increase of 51%, with a range
275 increase of 2–23% for the birds in a selected Gould & Heppner (1974) flock. Hainsworth (1987)
276 later provided an excellent description of the modified projective geometry technique he and
277 Badgerow used in the study above for examination of goose flocks, and applied it to his own
278 photographs of Canada Goose flocks. He noted that birds frequently shifted positions laterally
279 relative to the bird ahead, although the basic energy saving model of Lissaman & Schollenberger
280 (1970) predicted that there was an optimum position for energy saving. Using their model, he
281 concluded that the goose flocks he filmed were only enjoying a 36% energy advantage over
282 solitary flight, about half of the Lissaman & Schollenberger (1970) model. He cautioned against
283 a simplistic engineering model for explaining in toto a behaviour that might be highly variable,
284 depending on circumstance.

285 O'Malley & Evans (1982a, b) broadened the examination of line formation flight by studying
286 line flight in White Pelicans, *Pelecanus erythrorhynchos*. They used a variant of Gould &
287 Heppner's (1974) projective geometry technique to measure angles of Vs and Js, and distance
288 between birds, with much larger sample sizes (45 flocks) than in the Gould & Heppner (1974)
289 study. The angles ranged from 24–122°, with a mean \pm SD of $67\pm 8^\circ$, $N=12$, for V formations, and
290 $70\pm 5^\circ$, $N=33$, for J formations. As in the earlier goose formation measurements, there was wide

291 variation in the measured angles, and the means were well below Lissaman & Schollenberger's
292 (1970) predicted optimum angle of 120° for maximum aerodynamic advantage. Again, as in the
293 goose studies, V formations were less common than single line formations.

294 Hummel (1983), an aerodynamacist, further refined the theoretical aspects of formation flight
295 by considering wing shape, homogeneous vs. non-homogeneous spacing, size of bird, flight
296 speed, and straight vs. curved lines. He concluded that, under optimum conditions of the above,
297 energy savings for formation flight were possible due to aerodynamic considerations, but the
298 wide variance seen in the arrangements of flocks in the field suggested that aerodynamics might
299 not be the only factor in formation flight.

300 Badgerow (1988) took a fresh look at the aerodynamic and visual hypotheses, and tried to
301 organize the scant real field data in such a way that they could be subject to test. He suggested
302 that if aerodynamic advantage was the primary driver of line flight, there should be a certain
303 geometric relationship between birds in a formation, but if visual considerations were
304 paramount, there should be a different configuration. Unfortunately, the variation in data
305 between flocks was sufficiently large to prohibit a clear distinction between the hypotheses,
306 although Badgerow felt that there was a non-trivial (about 10%) energetic advantage of
307 formation flight over solo flight.

308 Cutts & Speakman (1994) also found wide variation in placement of individuals in their study
309 of formation flight of Pink-footed Geese, *Anser brachyrhynchus*. They photographed 54 skeins
310 from directly beneath, simplifying the extraction of distances and angles. They found that large
311 numbers of birds flew outboard of the position predicted by theory to maximize aerodynamic
312 savings, resulting in a postulated mean energy saving of 14%. Further, after a discussion of
313 optimum flight speed for optimum range, they suggested that if the birds in their sample flew at
314 a speed that would maximize their range, the savings would drop to 2% of that predicted by
315 Lissaman & Schollenberger (1970). Speakman & Banks (1998) later used the same technique to
316 photograph 25 formations of Greylag Geese, *Anser anser*. They found a great deal of variation in
317 positioning and that only 17% of birds flew in the predicted optimum position for aerodynamic

318 savings. They suggested, using the same assumptions as the Cutts & Speakman (1994) paper,
319 that the mean saving in induced power was 27%, and the reduction in total flight costs was
320 5–9% of the whole. Hainsworth (1988) also found in film studies of Brown Pelicans, *Pelicanus*
321 *occidentális*, that there was wide variation in wingtip spacing, and that there was no evidence
322 that the birds spaced to optimize possible aerodynamic effects.

323 Shortly after the turn of the new century, several papers appeared with a decidedly more
324 mathematical bent than had been seen previously, from investigators with backgrounds in the
325 control of multiple autonomous unmanned aerial vehicles, like the Predator, and Global Hawk.
326 Seiler et al. (2002) noted the wide variation in distances and angles reported in bird formations
327 in previous field studies, and in a rather puzzling table suggested that the average number of
328 birds in a V formation appeared to be small, typically under 10 birds. Other studies (Gould &
329 Heppner 1974; Hainsworth 1987) had reported mean V formation sizes closer to 20 birds.
330 Seiler et al. (2002, p. 122) noted that, on theoretical grounds, maintenance of a specific spacing
331 and angular relationship between a ‘leader’, and following autonomous robotic vehicles is a
332 daunting task, and that errors in spacing rapidly multiply with each subsequent vehicle, so
333 much so that ‘—flying in close formation is *not possible* (italics added) with information only
334 about the predecessors’. In other words, if a vehicle attempts to maintain position in the
335 formation only by maintaining position with its immediate predecessor in line, the formation
336 itself will quickly break down. However, they proposed two potential resolutions:

- 337 1. the formations should be very small, and/or
- 338 2. leader positional information should be simultaneously communicated to all
339 members of the formation; in other words, a trailing bird should maintain its position
340 with respect to the leader, rather than its immediate predecessors.

341 The same team expanded this idea, and explored the concept of ‘string instability’, the
342 phenomenon where the trailing vehicle in a line has such difficulty tracking predecessors that it
343 oscillates in position to such a degree that it eventually cannot stay with the formation (Seiler
344 et al. 2003). In particular, they explored the difficulties of maintaining lateral positioning in a

345 line formation. They proposed that the difficulty in maintenance of position increases markedly
346 with position back from the leader; the 'positional error' (assuming the birds were 'trying' to
347 maintain an optimum position for either aerodynamic or visual reasons) of the number four
348 bird in relation to the lead bird would be twice as much as that of the number two bird. Seiler
349 et al. (2003, p. 279) concluded by suggesting '—that (avian) formation flight is *inherently* (italics
350 added) difficult'. A glance overhead at a winter waterfowl assembly area displaying a panorama
351 of dozens of birds flying in each of hundreds of separate line flocks suggests a variant of the
352 catch phrase of the late Spanish ventriloquist, Señor Wences, 'Difficult for me; easy for you'.
353 Seiler et al. (2003) suggested that their hypothesis could be tested by examining whether birds
354 further back in the formation have a greater variation in wingtip spacing than those closer to
355 the leader. This hypothesis, of course, rests on the prior hypothesis that there is an optimum
356 spacing that the birds are attempting to maintain.

357 Weimerskirch et al. (2001) have provided the best (and to date) most realistic attempt to
358 resolve in the field whether there is an energy advantage to line formation flight. They trained a
359 flock of eight Great White Pelicans, *Pelecanus onocrotalus*, to fly in formation behind a
360 motorboat. Energy consumption during flight was not recorded directly, but inferred from heart
361 rate data. They measured heart rate from selected individuals in the flock, and from a solitary
362 bird flying under the same conditions. Heart rates of the birds in formation were 11–15% lower
363 than that of the solitary bird. From this, they concluded that they had provided empirical
364 evidence of an aerodynamic advantage to formation flight, in about the same fractional
365 proportion as the heart rate difference.

366 An alternate interpretation of the data is possible, especially given the relatively scant
367 proposed saving compared to most aerodynamic theory-based predictions. Pelicans are highly
368 social animals and the experience of flying solo might have been stressful compared to normal
369 social flight. Späni et al. (2003) found that laboratory mice housed individually had a heart rate
370 4% higher than that of mice housed in pairs. So the effect seen may have been due, at least in
371 part, to social stress rather than aerodynamic advantage.

372 Modelling, Simulations and Application

373 The development of very powerful, relatively inexpensive computers in the late 1990s
374 permitted a more sophisticated mathematical analysis of V formations. The first to report a
375 model producing V formations was Flake (2000, pp. 270–275), who extended Reynolds' (1987)
376 model (to be discussed later) with an additional rule; each artificial bird, or 'animat' (Wilson
377 1985; Watts 1998), 'attempted' to move laterally away from any animat that blocked its view to
378 the front, and with that achieved V-formation flocks.

379 Assuming that there is, in fact, a reduction in collective aerodynamic drag experienced by
380 members of a flock in a V, Dimock & Selig (2003) went a step further and developed a computer
381 simulation that actually modelled the induced drag. They extended Reynolds' (1987) model to
382 'detect' potential drag reductions by adding a rule by which each animat acted to reduce the
383 drag, and observed how the animats self-organized themselves. There was an evolutionary
384 component to this study—they used genetic algorithms to evolve the model's parameters and as
385 each animat acted so as to reduce its own drag, the collective result was that the drag reduction
386 of the flock as a whole was maximized. Limiting the utility of the model, their induced drag
387 calculations were based on a rigid wing just as Lissaman & Schollenberger's (1970). In relatively
388 short simulations, their model correctly penalized collisions, and ultimately produced
389 rigid/stable flocks of perfect Vs. Using the same evolutionary theme, Andersson & Wallander
390 (2004) suggested that kin selection might explain why there appeared to be so much variation
391 in V formation structure. Most aerodynamic advantage studies propose that the lead position is
392 to some degree less advantageous than following positions, but Andersson & Wallander (2004)
393 suggested that if the flock is composed of kin, the leader might enjoy a gain in inclusive fitness,
394 even if at a personal energetic disadvantage. A casual glance at feeding or migrating flocks
395 suggests considerable shifting of position, and 'leadership' changes within the flock, but it would
396 be useful if there were a quantitative study indicating whether all or most birds assume the
397 'leader' position during a flight.

398 Nathan & Barbosa (2008) developed a comprehensive computer model that produced V
399 formations. Their model evolved from a series of simulations that yielded cluster flocks
400 (discussed below). The animats in their model followed simple rules; each bird attempted to
401 seek the proximity of the nearest bird (while avoiding collision), each bird attempted to find a
402 position that offered an unobstructed longitudinal view (if the first rule was not applicable), and
403 each bird attempted to position itself in the upwash of a leading bird. Using these rules, they
404 were able to produce Vs, Js and echelons; as well as inverted Vs which are rarely seen in nature.
405 The model was limited in its ability to handle flock turning movements as it assumed a constant
406 heading and the rules produced only lateral displacements. An attractive feature of the model
407 was, nonetheless, that it offered the opportunity to test the relative importance of aerodynamic,
408 or communication hypotheses, by changing the values of parameters.

409 So, Why do Birds Fly in a V Formation?

410 After over 30 years of active interest in the field, we may be reasonably certain of the following
411 things;

- 412 1. Many large birds (but not all) fly in line formations; small birds almost never.
- 413 2. The V and J formations are the most striking and eye catching line formations for
414 humans to observe, but they are not the most common for birds to fly in; the echelon
415 has that distinction.
- 416 3. There is wide variation, from flock to flock and species to species, in positioning and
417 distances of individual birds in a line. Aerodynamic theory predicts, however, that
418 there is an optimum position and distance between birds if aerodynamic advantage is
419 to be maximized, both for individuals and flocks.
- 420 4. The lines are wavy as often as they are straight.

421 One of us (FH) once asked a WWII B-17 pilot why bombers flew in a V. His reply was, 'To keep
422 a clear field of fire for the guns to the front, and to keep an eye on the leader, who does the
423 navigation.' Birds clearly need not worry about the former, but in fact the 'leader' is

424 determining the direction the flock is to take, it would be an advantage to keep it in sight, an
425 advantage in a large flock accruing to a curved or irregular line.

426 Why not fly directly to the side of the leader, or directly in back? If a bird flew to the
427 immediate left or right of another bird, a gust of wind or a startled response from the neighbour
428 might precipitate a collision. Similarly, if the bird ahead were to suddenly slow down for any
429 reason, a rear-end collision might be possible. On an uncrowded motorway, drivers rarely
430 prefer to drive for long distances alongside a car in an adjacent lane, or tuck in close behind a
431 leading car if there is an opportunity to pass, possibly for similar anti-collision reasons. If the
432 object of the staggered line formation is primarily to avoid collision while keeping a leader in
433 sight, one would expect to see wide variation in spacing and alignment, simply because there is
434 no particular advantage to one spatial relationship rather than another. Similarly, one would
435 expect to see undulations in the line. As the body of a neighbour momentarily blocked the view
436 of the leader, perhaps due to a wind gust, an individual bird could simply speed up a bit or drop
437 back to regain sight of the leader, thus precipitating a wave.

438 But what of the potential aerodynamic advantage of V flight? Aerodynamic theory suggests
439 that one exists, under certain conditions. One must ask about its relative importance and need,
440 however, as it is noted that most of the field studies of line formations have not been made on
441 migration flights, where energy savings, even small ones, might well be of importance, but on
442 short feeding flights of 10–20 km, where the energy expended in flight represents a small
443 fraction of the birds' daily energy budget, and that whereas staggered lines are common, Vs and
444 Js are much less so. We simply do not know what kinds of formations large birds use on their
445 long migratory flights, which are often over water. Additionally, there may be an energetic cost
446 to flying in close formation. The stress level in flying in very close proximity to other birds, with
447 consequent collision risk, might (on migration flights) raise metabolic levels enough to partially
448 negate any aerodynamic energy advantage of close formation flight.

449 The 'crucial' experiment, to determine if, in formation flight, there is a worthwhile energy
450 advantage to be gained for aerodynamic reasons, might be to train a group of imprinted line-

451 formation birds like geese to fly in a wind tunnel, and then use modern airflow visualization
452 techniques to empirically determine what the upwash properties of birds flying in formation
453 really are (Pennycuick et al. 1997; Rayner 1995).

454 **CLUSTER FLOCKS**

455 There is an extensive literature discussing the biological value of flocking in general (Krebs &
456 Barnard 1980), but very few papers have appeared with specific reference to the highly
457 organized turning and wheeling ('cluster') flocks of some small birds. The most commonly
458 offered hypothesis is that the closely spaced cluster flocks offer protection against aerial
459 predators like hawks, presumably by increasing the risk of collision to the predator (Tinbergen
460 1953). Examples have been reported where flocks of starlings and shorebirds bunch up tightly
461 when attacked by a hawk (Major & Dill 1978). This hypothesis appears reasonable, but leaves a
462 commonly seen behaviour in some cluster flying species to be explained. At sunset, or just
463 before, large flocks of European Starlings will form over a roost from smaller foraging flocks
464 that have dispersed during the day from that roost. These flocks will engage in some of the most
465 spectacular group movements seen in flocking birds for periods of 30–45 min before settling
466 into the roost for the night. Two questions immediately present themselves: 1) Do not these
467 movements 'waste' energy in species for which energy is important (Hamilton et al. 1967)? and
468 2) by occurring every night in the same location, and being highly visible from up to a km away,
469 do they not almost invite predator attack? A loitering predator would have an excellent
470 opportunity to pick off a straggler (we have seen many pre-roosting turning and wheeling flocks
471 that generate stragglers as the flock splits and rejoins). Why do these flocks not land
472 immediately in the roost after returning from foraging, and why are there often 10–15 min of
473 coordinated turning and wheeling before a flock descends to a feeding area, both expending
474 energy, and facilitating predation?

475 Wynne-Edwards (1962) proposed instead that these movements represented 'epideictic'
476 displays that might enable individual flock members to assess the population numbers and

477 density of the flock as a whole; information that might be used in regulating breeding behaviour.
478 This suggestion was part of a larger concept that is called today 'naïve group selection'. This
479 hypothesis received little support at the time (Crook 1965), but has enjoyed a recent re-
480 examination (Wilson & Wilson 2007) that may be useful in considering how organized flight
481 evolved. Another hypothesis was provided by Major & Dill (1978) who suggested that these
482 turning and wheeling movements were 'protean' (Driver & Humphries 1970); irregular
483 movements designed to confuse potential predators. A number of recent studies (Biro et al.
484 2006; Codling et al. 2007; Dell'Arciccia et al. 2008) have suggested that flying in a group
485 improves homing performance in pigeons, but it is not clear that the structure of the flock has
486 anything to do with this improvement.

487 There are several questions that are usually asked when considering the mechanism, rather
488 than the function of cluster flocks:

- 489 1. Do the flock members truly turn simultaneously during a turning movement, or is
490 there a wave of movement starting at a centre somewhere and passing through the
491 flock?
- 492 2. Is there a leader in the flock who communicates an action intention in some fashion to
493 other members of the flock, or is there some emergent property of flocking itself that
494 produces coordinated movement?
- 495 3. What mechanism governs the departure of flocks from the roost, ground, or perch, in
496 which sometimes the whole flock departs, and at other times, subgroups will depart
497 before the main group?

498 Simultaneity (or not) of individuals making a turn has relevance to the related question of
499 leadership. A wave of turning in the flock would suggest (but does not necessarily provide
500 evidence for) a relatively simple model where a leader turns, followed by a turn by neighbours
501 after a suitable reaction time (which was established by Pomeroy & Heppner [1977] to be under
502 100 ms in laboratory studies of starlings), then a wave passing through the flock as birds
503 respond to a turn by birds distant from the leader, but who ultimately have responded to a turn

504 initiated by the leader. Vision would be the most parsimonious medium for information
505 transmittal in such a model. If birds turn simultaneously instead (within the limits of the
506 recording instrumentation), the question becomes more interesting; either a putative leader has
507 to communicate a message instantaneously to all members of the flock, seemingly ruling out
508 sound and vision in large flocks (because the bodies of nearby neighbours would block the view
509 of more distant birds), or it would be necessary to propose an organizing principle that could
510 produce synchronized turns without leadership. Such a model only became available in the
511 1980s.

512 Davis (1980) filmed turning flocks of Dunlin, *Calidris alpina*, with a slow-motion cine camera
513 (72 frames/s). Dunlin are differentially coloured on their dorsal and ventral surfaces, and Davis
514 observed that some individuals in flocks of approximately 40 birds all appeared to turn within
515 120 ms, giving the appearance of a 'flash'. Potts (1984) using a similar technique with Dunlin,
516 noted some examples of waves of turning that propagated from neighbour to neighbour within
517 14 ms, considerably faster than the measured reaction times in birds. He proposed a 'chorus-
518 line' hypothesis to account for rapid turns, in which one bird or a small group could initiate the
519 movement, which would then be followed by neighbours who responded to their immediate
520 neighbours and whose speed of response would depend on their own reaction times, but more
521 distant birds would be able to estimate and anticipate the passage of the wave, as in the
522 'Mexican wave' in stadiums (Farkas et al. 2002). However, Heppner (1997) suggested the
523 possibility that a perceived wave of turning in differentially turning birds might be an artefact of
524 observer position relative to the near and far borders of the flock, and that individuals in a flock
525 apparently turning in wave fashion might in fact be turning nearly simultaneously.

526 Early on it was realized that to approach these questions, some idea of the geometric
527 relationship between birds in a cluster flock would be needed, and that meant the development
528 of three-dimensional (3D) analysis techniques. These techniques are well developed for
529 laboratory studies of fish schools (Partridge et al. 1980), but are much more challenging for
530 birds in the field.

531 Major & Dill (1978) obtained the first 3D measurements of distances between birds in free-
532 flying flocks of Dunlin, *Caladris alpina*, and European Starlings by using a stereoscopic
533 photographic technique that utilized two 35 mm film cameras whose optic axes were parallel,
534 and which were firmly fixed on an aluminium bar 5.5 m long. They were particularly interested
535 in nearest-neighbour distances, and the angles between neighbours, as these would provide an
536 index of condensation of the flocks. They reported that the nearest neighbour to a reference
537 bird was typically behind and below a reference bird, a pattern often seen in fish shoals.

538 Pomeroy (1983) and Pomeroy & Heppner (1992) used an orthogonal 3D photographic
539 technique to obtain sequence pictures of semi-domestic Rock Pigeons, *Columba livia*, turning in
540 flocks of 8–11 birds. Using this technique, they were able to plot the flight paths of individual
541 birds, as well as nearest neighbour distances. They reported that the flight paths of individual
542 birds crossed over each other, such that in a 90° turn, a bird that had been in the lead would be
543 to the right or left of the flock, and after a 180° turn, would be in the rear of the flock. They
544 suggested that an individual bird would find it difficult to ‘lead’ a flock by positioning itself at
545 the head of the flock.

546 Ballerini et al. (2008a, b) and Cavagna et al. (2008a, b) have developed a powerful tool for the
547 analysis of cluster flocks by essentially solving the ‘correspondence problem’ that has bedevilled
548 photographic 3D analysis techniques. Most such methods involve taking a pair of pictures from
549 slightly different viewpoints, and noting the displacement of the image of a single bird in one
550 view from the other member of the pair. With large numbers of birds of identical appearance,
551 how do you match the image of the same bird in the two views? By using a novel statistical
552 method, they were able to determine the positional relationships of over 1,000 birds in a
553 European Starling flock flying over Rome. Using data obtained by this technique, Ballerini et al.
554 (2008a) suggest that the significant factor in determining interaction between birds in cluster
555 flocks is not the distance between birds (‘metric’ distance), but the number of birds between
556 any two birds (‘topological’ distance, in their terminology).

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558 Davis (1980), after reviewing the deficiencies of a leadership model for cluster flock turning and
559 wheeling movements, suggested the possibility that a 'self-generated synchronous activity'
560 might provide a model for coordinated movements. Within a decade, the development of
561 accessible and powerful computers and programming languages produced such models.
562 Working independently, Okubo (1986), Reynolds (1987) and Heppner & Grenander (1990) each
563 developed flock flight models based on the concept that each bird in a flock followed simple
564 behavioural rules in relation to its neighbours, and that the interaction of these rules produced
565 the emergent property of a coordinated flock. Moreover, Okubo (1986) and Reynolds (1987)
566 suggested that the same concept could be employed to model schools and herds, which lead to
567 numerous studies in all three fields based on similar models, as if to find a universal theory. To
568 some extent the conceptual ancestor of all models was John Conway's 'Game of Life', (Gardner
569 1970), one of the first cellular automata that demonstrated how complex global behaviours can
570 arise as a product of self-organization by simple components following simple local rules.

571 To be specific, Reynolds (1987) proposed that for the purpose of computer animation a flock
572 could be modelled as a group of animats (or 'boids', using his terminology) that followed three
573 simple rules, which might behaviourally be rephrased as 'drives'. These drives caused the
574 animats to attempt to avoid collisions with nearby neighbours ('separation' or 'repulsion'),
575 match velocity with nearby neighbours ('velocity matching'), and stay close to nearby
576 neighbours ('cohesion' or 'attraction'). The term 'nearby' was used to describe the animat's
577 localized perception of the universe. In all incarnations of the model, Reynolds (1987, 1999,
578 2004) used drive dependent perception volumes (nearby neighbours were all animats within a
579 sphere of a predefined diameter centred at the currently observed animat's origin) with a
580 biologically realistic perception model (limitations of visual perception were accounted for; a
581 'blind cone' [the three-dimensional equivalent of a 'blind spot'] was subtracted from the
582 perceptual sphere at the back of the observed animat). At the time of proposal the approach
583 represented a giant step forward compared to the traditional techniques used in computer

584 animation for motion pictures. The first animation created with the model was 1987's '(Stanley
585 and Stella in) Breaking the Ice', followed by a feature film debut in Tim Burton's 1992 film
586 'Batman Returns' with computer generated bat swarms and 'armies' of penguins marching
587 through the streets of Gotham City. The current state of the art in computer animation for
588 motion pictures has evolved even further (Massive 2008); these advanced models, however, due
589 to the obvious financial consequences, remain proprietary.

590 Heppner & Grenander's (1990) distinguishing features were the approach used to model
591 perception and the animats' drives. In their case the same perception volume (a sphere of a
592 predefined diameter centred at the currently observed animat's origin) was used for all drives
593 and limitations of visual perception were not accounted for. The animats attempted to stay in
594 the roosting area ('homing'), attempted to fly with a predefined flight speed ('velocity
595 regulation'), and attempted to move apart if too close, or closer if not too distant ('interaction').
596 An additional feature was 'random impact', which was intended to simulate the random
597 distractions that are present in a natural environment (wind gusts, distractions from moving
598 objects on the ground, etc.). Heppner & Grenander (1990) implemented the latter by using a
599 Poisson stochastic process and admitted that without its inclusion they were unable to produce
600 a flock-like behaviour.

601 In the mid 1980s and early 1990s, computer processing power was limited and real time
602 simulations of large flocks consisting of more than a few dozen birds were infeasible. The first
603 step to simulations that would allow observing an animated output while running the
604 simulation and interactively changing the model's parameters was performed by Lorek & White
605 (1993), who, just a few years after Reynolds' (1987) paper was published, used a Meiko
606 Transputer System with up to 50 processors to run flight flock simulations, consisting of merely
607 100 birds at slow, but interactive rates (6 frames/s). The recent advances in multicore
608 technology (Gschwind et al. 2006) and computer graphics dedicated hardware (NVIDIA 2007),
609 and their use for scientific research (Khanna 2007; Sijbers & Batenburg 2008) give the
610 impression that barriers to real time simulations and interactivity will soon be breached.

611 Reynolds (2006), for example, reported a multicore solution, which takes advantage of the Sony
612 PlayStation 3 Cell processor for running simulations of 10,000 fish with animated output of
613 cinematic quality at 60 frames/s. More recently, at the SIGGRAPH 2008 conference, the Game
614 Computing Applications Group of AMD, Inc. was showing a cinematic quality technology demo
615 titled 'March of the Froblins' (AMD 2008; Shopf et al. 2008), a graphics processing unit (GPU)
616 based crowd simulation of 65,000 agents at 30 frames/s.

617 The techniques used for achieving high frame rates might at times be at the expense of
618 biological realism. It is also true that for a scientific study centred on behaviour, the ability to
619 interactively change the model's parameters and observing the effects in real time is a welcome
620 plus. In computer animation for games and virtual reality (Brogan et al. 1998) high frame rates
621 are important and the modelling of flocking behaviour has a niche of its own—it falls under the
622 subject of 'controlling groups of objects'. Disregarding the cost of achieving the desired degree
623 of visual realism, the simplicity of achieving high frame rates depends on the class of the
624 controlled group of objects. Using Parent's (2002) terminology, there are three principal classes
625 of controlled groups: 1) 'particles', characterized as large collections of individual objects, each
626 of which obeys simple physical laws, such as momentum and conservation of energy, but has no
627 'intelligence', or decision making capacity; usually, such particles interact mostly with their
628 environment, and there is little, if any, inter-individual exchange (typical examples are models
629 of fluids, gaseous phenomena, hair, fur, etc.); 2) 'flocks', characterized as medium (fewer in
630 number than particles) size collections of individual objects, with some incorporated physics
631 and intelligence—interaction with the environment and inter-individual exchange (typical
632 examples are models of schools, swarms, herds, crowds, traffic, etc.); 3) 'autonomous agents',
633 characterized as small collections, with little, if any, incorporated physics and much intelligence
634 (typical examples are intelligent agents, autonomous robots, software agents, computer viruses,
635 etc.). All three classes are examples of independently behaving members of groups with varying
636 levels of autonomy, physical characteristics and simulated motions.

637 Kennedy & Eberhart (1995) were the first to incorporate elements from artificial life and
638 artificial intelligence (AI) studies to allow bird flocking behaviour models to serve as exemplars
639 for more general kinds of behaviour, including human social behaviour (Helbing & Molnár
640 1995). A group of interacting animats is a 'swarm' in AI terms, and Kennedy & Eberhart (1995)
641 presented algorithms by which a swarm might optimize its behaviour, or adapt to serve some
642 end, such as increasing energy input. The animats in a swarm make decisions about their own
643 behaviour based on the behaviour and knowledge gained from their neighbours, as well as the
644 perceived elements of their surroundings, such as locations of 'feeding areas'. From the swarm's
645 collective behaviour emerges the animats' indirect approach to relatively-good solutions. These
646 algorithms are also known as 'particle swarm optimization', or PSO for short (Kennedy et al.
647 2001; Engelbrecht 2006). Macgill & Openshaw (1998) and Macgill (2000), for example, later on
648 used flocking behaviour to assist the analysis of geographical data. Subsequent AI studies
649 started introducing more and more intelligence in individual animats while reducing their
650 number; Odell (1998) provided a summary of the terminology and properties attributed to
651 'agents' in computer studies.

652 In the mid-1990s, physicists began to show interest in the mathematics and physics of
653 organized flocks, using a perspective very different from those found in earlier biological and
654 aerodynamic studies. Vicsek et al. (1995) and Toner & Tu (1995, 1998) viewed the birds in a
655 flock as particles, behaving much as molecules in a fluid or atoms in a crystal might, and that
656 they were responsive to the same mathematical rules. The models were all based around the
657 same perception model as in Heppner & Grenander (1990); inter-individual influences occurred
658 between animats that were not further apart than a predefined distance. Additionally, an overall
659 constant flight speed was assumed and the number of drives was reduced to merely one,
660 attempting to match flight direction with nearby neighbours. A stochastic component had been
661 added in these models, perhaps on similar presumptions as the random impact used by
662 Heppner & Grenander (1990). As theoretical physicists, Toner & Tu (1998, p. 4830) may have
663 been somewhat removed from realities in the field when they suggested, 'This correlation

664 function should be *extremely easy* (italics added) to measure in simulations, and in experiments
665 on real herds or flocks, in which, say, video tape allows one to measure the positions ... of all the
666 birds ... in the flock at a variety of times t' . Vicsek et al. (1995) suggested that the physics
667 concepts associated with phase transitions, as in the transition from solid to liquid forms of
668 materials, might serve to explain the puzzling shifts between orderly and disorderly flock
669 formations often seen in birds like European Starlings. All in all, their models represent a
670 substantial simplification in biological assumptions over the initial ones proposed by Okubo
671 (1986), Reynolds (1987) and Heppner & Grenander (1990). Using Parent's (2002) terminology,
672 these models fall perfectly under the particles category. The model devised by Vicsek et al.
673 (1995) is also known as the 'self-propelled particles' model, or SPP. Physicists embraced these
674 minimalist models and a number of subsequent studies have been published (see Czirók et al.
675 1997; Czirók & Vicsek 1999, 2000; Tu 2000; Li et al. 2007; Li & Xi 2008; Chaté et al. 2008; Gönci
676 et al. 2008; Huepe & Aldan 2008, for example). Recent field observations by Ballerini et al.
677 (2008a, b) and Cavagna et al. (2008a, b) seem to be making an impact in the physics community
678 and even physicists are starting to acknowledge the importance of inclusion of attractive-
679 repulsive drives (Grégoire et al. 2003; Grégoire & Chaté 2004; Feder 2007). An additional result
680 of these field observations, is one that somewhat contradicts the approach commonly assumed
681 by flock flight models. These typically assume a fixed radius of interaction. Data obtained by
682 Ballerini et al. (2008a, b) and Cavagna et al. (2008a, b) seems, on the other hand, to suggest that
683 it is not the radius, but the number of influencing individuals, which remains constant.

684 Just as Kennedy & Eberhart (1995) incorporated AI elements into flight flock models to
685 devise PSO, other computer science studies applied AI algorithms to evolve the models
686 themselves. Reynolds (1993a, b), Zaera et al. (1996), and Spector et al. (2005), used genetic
687 programming, a technique for automatically creating computer programs that satisfy a specified
688 fitness criterion, to evolve the individual animat's rules, or in this case 'programs', which, when
689 the animats interacted, produced flocking behaviour. All previous models employed constants
690 like 'perception radius' (diameter of the sphere centred at the currently observed animat's

691 origin, by the use of which nearby neighbours are selected), ‘weights’ (typically the direction of
692 flight [and flight speed] of the currently observed animat is computed as a weighted sum of the
693 individual ‘desired’ flight directions that would meet the individual drives, respectively), etc.
694 Heppner & Grenander’s (1990) approach was to modify these by hand and analyse the results.
695 The AI approach was to use evolutionary computing. Genetic algorithms were used to vary the
696 parameters to optimize the behaviour to a specified fitness criterion. Dimock & Selig (2003), for
697 example, used genetic algorithms on a modified Reynolds (1987) model to find parameters for
698 minimum power consumption in a flock of simulated birds. Wood & Ackland (2007), on the
699 other hand, using a Couzin et al. (2002) model, studied the evolution of group formation when
700 subjected to simulated predation and foraging. Their results replicate conventional evolutionary
701 behaviour—foraging animats prefer a narrower perception volume, while the hunted prefer a
702 wider one.

703 Couzin et al. (2002) and Couzin & Krause (2003) added the next level of sophistication in
704 flocking models. The substantial difference was not in the animats’ drives, but in the perception
705 model, or when these drives were actually in effect. Whereas Heppner & Grenander (1990) used
706 one perception volume for all three drives, Reynolds (1987, 1999, 2004) three non-exclusive
707 perception volumes with biologically inspired limitations, Couzin et al. (2002) and Couzin &
708 Krause (2003) introduced a different approach; in their model there were three exclusive
709 perception volumes, or ‘zones’, using their terminology: 1) ‘zone of repulsion’, 2) ‘zone of
710 orientation’ and 3) ‘zone of attraction’. If there were neighbours in the zone of repulsion, then
711 only the separation drive was active and the other two ignored. If, however, there were no
712 neighbours in it, the other two drives were averaged, but the animat attempted to match
713 velocity only with the neighbours in the zone of orientation and attempted to stay close only to
714 the neighbours in the zone of attraction. Additionally, the zone of repulsion was modelled as a
715 sphere, whereas the other two were modelled as a sphere with a blind cone subtracted at the
716 animats’ back. Couzin et al. (2002) considered what would happen to group movements if
717 individuals in the group modified their behavioural rules in response to experience with the

718 flock as a whole. More specifically, what would happen if the diameter of the zone of orientation
719 was variable, while keeping the zones of repulsion and attraction constant. They found that, as
720 the diameter of the zone of orientation increased, the group went from a loosely packed
721 stationary swarm, to a torus where individuals circle round their centre of mass and, finally, to a
722 parallel group moving in a common direction (see also Sumpter 2006). Further on they
723 discovered that the transitions were rapid, and as the diameter decreased, the collective
724 behaviour was different. They established that two completely different behavioural states can
725 exist for identical parameters, and that transition between behavioural states depends on the
726 previous history (structure) of the group, even though the individuals have no explicit
727 knowledge of what that history is. Consequently, they suggested that the system exhibits a form
728 of 'collective memory'. In a later study, Couzin et al. (2005) examined leadership and decision
729 making in animal groups on the move by giving knowledge of a preferred flight direction only to
730 a proportion of the simulated animals. The study revealed that the larger the group, the smaller
731 the proportion of informed individuals needed to guide the group, and that only a small
732 proportion is required to achieve great accuracy. Several recent experimental studies (Biro et al.
733 2006; Codling et al. 2007; Dell'Arciccia et al. 2008) investigated the 'many-wrongs principle' in
734 pigeon homing and suggest that pigeons flying in a group have better navigational performance
735 than birds flying alone, but it is not clear whether the spatial organization of the flock is
736 significant in this observation.

737 Lebar Bajec et al. (2003a, b, 2005) and Lebar Bajec (2005) introduced the concept of fuzzy
738 logic to flocking models. The basic concept of the model remained the same; three drives and
739 perception modelled as a sphere with a blind cone removed from the back. But in previous
740 models, the animats would react to their surroundings in a 'crisp' way. For example, if we are
741 interested in two moving animats that are on a closing course with one another, there might be
742 some specific threshold distance at which they would deviate to avoid collision (e.g. when they
743 enter each other's zone of repulsion [Couzin et al. 2002]). Or, in a slightly more complex
744 example, there might be a gradient for different closing angles such that the animats would

745 deviate proportionately, but still in deterministic fashion depending on the closing angle.
746 However, with fuzzy logic, vague qualities like ‘close’ or ‘far’ rather than a specific distance or
747 angle can be used to describe the behavioural repertoire of the animat. In this fashion, a more
748 naturalistic type of behaviour can be produced. Indeed, Heppner & Grenander (1990) used a
749 single perception volume, Reynolds (1987, 1999, 2004) three overlapping perception volumes,
750 and Couzin et al. (2002) and Couzin & Krause (2003) advanced the model by introducing three
751 non-overlapping perception zones, the use of vague qualities enabled Lebar Bajec et al. (2005)
752 to produce a mixture of these approaches with partially overlapping perception zones. The issue
753 this model has, with respect to the others, is that it is two dimensional; animats can move left or
754 right, but not up or down. As real birds exist in three dimensions, a genuinely realistic
755 simulation needs to feature the third dimension. Moškon et al. (2007) expanded the fuzzy model
756 to account for foraging behaviour by including hunger as a drive. While doing so, they also
757 modelled foraging fields and landing and taking off from them; while this has not been achieved
758 by promoting the drives to work in three dimensions, they upgraded the model to pseudo 3D
759 nonetheless.

760 So, How do Birds Seem to Turn and Wheel Together?

761 In the 1970s, there was no conceptual alternative to a leadership model for producing
762 simultaneous or near-simultaneous turning movements in cluster flocks. With the advent of the
763 many models that treat flocks as collections of independently acting agents that produce turning
764 movements as the product of individual movement decisions, a viable alternative to leadership
765 models now exists, but such models 1) do not rule out the possibility that under certain
766 circumstances, particularly with small, or family flocks, leadership might still play a role in
767 cluster flock movements, and 2) do not provide evidence that birds use the same algorithms as
768 the models. Just as there may be several biological functions for line formations, it may be that
769 there are multiple mechanisms for producing cluster flock movements.

770 **CONCLUSION**

771 Advances in the understanding of the function and mechanisms of organized flight have been
772 strongly linked to the introduction of new techniques or technologies. Heppner (1997)
773 identified several areas that might be expected to produce such advances, but a decade later,
774 although it has been possible to refine and more closely define these needs, much still needs to
775 be done.

- 776 1. Three dimensional simulations. Some of the existing simulations (Vicsek et al. 1995;
777 Lebar Bajec et al. 2005; Mořkon et al. 2007; Nathan & Barbosa 2008), although
778 capable of producing realistic-appearing flocks on a computer screen, feature animats
779 that travel in a two dimensional universe. They may travel left or right, but not up or
780 down. As real birds exist in a 3D world, a genuinely realistic simulation would have to
781 feature the third dimension. Adding the additional dimension is not a trivial
782 programming task, but its accomplishment could be expected to pay large dividends.
- 783 2. Non-homogeneous models. To date, flight flock models have assumed that flocks are
784 composed of identical subjects. In reality, there will be individual differences in age,
785 gender, sensitivity to hunger, health, and other factors that may well influence the
786 collective behaviour of the flock.
- 787 3. Fast, cheap, field data acquisition. Cavagna et al.'s (2008a) technique for obtaining the
788 3D positions of thousands of birds in a flock has yielded remarkable results, but the
789 method requires custom-made synchronizing equipment for the cameras, skilled
790 operators, lengthy processing, and a fixed location. As a result, it is difficult to
791 compare species, conditions, or fine structure over time. The current generation of
792 digital still and video cameras offers the potential for both high resolution and a high
793 frame rate at a reasonable cost. Commercial wireless technology, such as that used to
794 simultaneously fire multiple remote flash units, offers the potential of synchronizing

795 two (or more) cameras in the field without the necessity for custom made
796 synchronizing devices.

797 4. User-friendly simulations. The current generation of flocking simulations is primarily
798 designed to be used and manipulated by their designers, who may or may not be
799 familiar with the behaviour of animals in the field. The programs are not easily used
800 or modified by other users unfamiliar with programming. It would be very helpful if
801 future simulations came with a 'console', or control panel that would allow non-
802 programmers to change the parameters or their values in the simulation, such as
803 preferred velocity, or attractiveness of feeding site, thus allowing field biologists to
804 examine the results of changing inputs to the program based on their field experience.
805 It might also be possible to set up 'detectors' in the program, as is done in
806 experimental particle physics, to allow many different combinations of parameters
807 and values to be run in sequence, and the program would flag interesting behaviours,
808 such as the appearance of a V, when they appear. For example, the Boston Museum of
809 Science in Massachusetts has a large public display called the 'Virtual Fishtank'
810 (Nearlife, Inc. 2001) that enables visitors to interactively change the behaviour of
811 individual fish in a 'school', and immediately see the change in the behaviour of the
812 school.

813 5. Metrics for 'truth testing'. Current simulations offer naturalistic appearing virtual
814 flocks, but it cannot be certain that real birds use the same algorithms employed in
815 the simulation. Ideally, one would produce a simulation of a particular species'
816 flocking behaviour, and use it to make predictions about the behaviour of the real
817 flock, and then test those predictions in the field. To do this, one would have to have a
818 metric that could be derived from the simulation, and then measured in the field. For
819 example, some simulations produce flocks that apparently turn and wheel much like
820 real flocks. Perhaps 'turning and wheeling' could be quantified, such that one could
821 say that, for example, a flock of X number of birds of species Y will make a turn,

822 defined as a departure of more than 20° from the mean direction exhibited in the
823 previous 5 s, every 8.2 s. If this variable were measurable in the field, it could then be
824 possible to refine the model to produce more accurate predictions. Successful
825 prediction would, of course, not be prima facie evidence that the algorithms in the real
826 and virtual worlds were the same, but would certainly provide stronger evidence than
827 a superficial, qualitative similarity. Dill et al. (1997) discussed this issue more
828 extensively.

829 The last 40 years have seen remarkable progress in the understanding of this intriguing and
830 aesthetically spectacular phenomenon. In addition to being a phenomenon worthy of
831 examination in its own right, the study of organized flight in birds has provided a model system
832 that has demonstrated utility in the study of crowd behaviour, bird strikes on aircraft, traffic
833 theory, complex systems, particle swarms, computer animation, and control of (remotely
834 piloted) autonomous aircraft. At this time, it is possible to foresee that with the assistance of
835 biologists, physicists, mathematicians and computer scientists working together, we will, before
836 long, truly be able to say how and why birds fly in organized groups.

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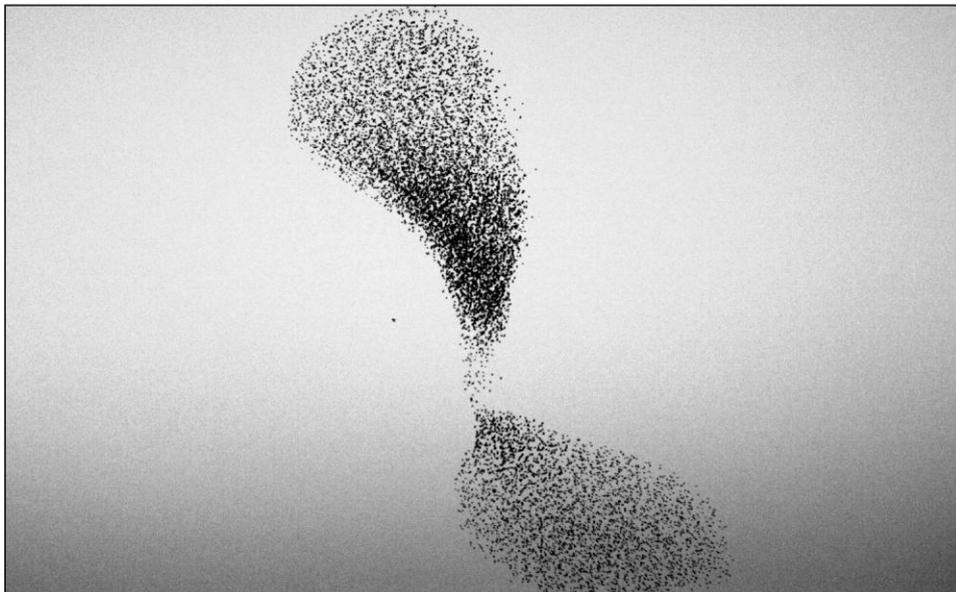
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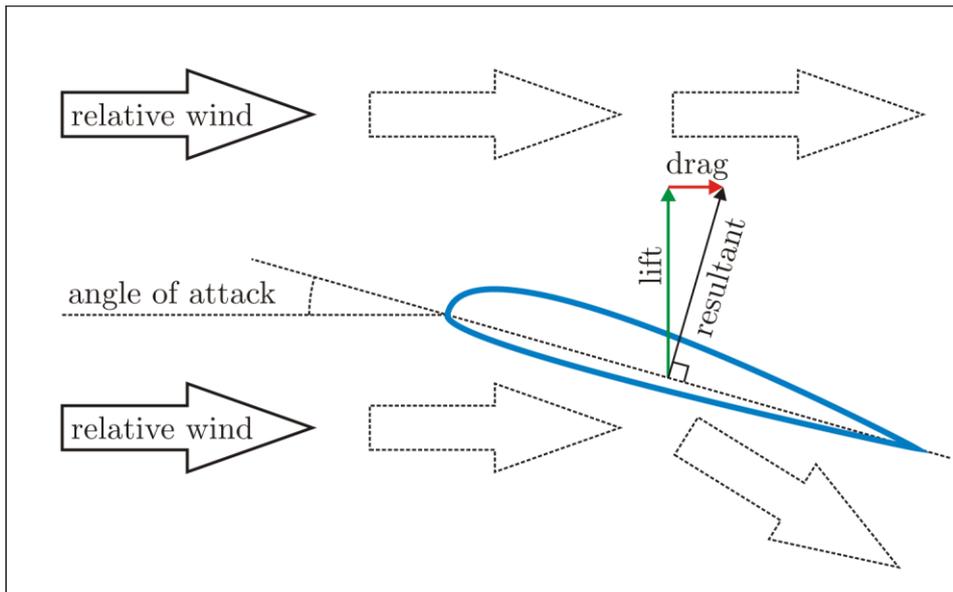


A) Line formation

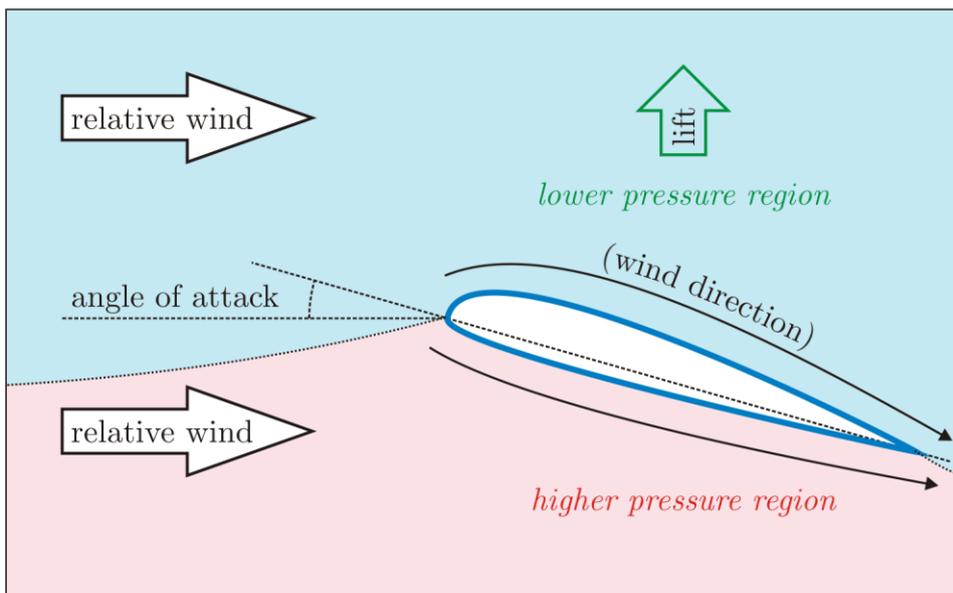


B) Cluster formation

1154 **Figure 1.** A) Line formation of Snow Geese, *Chen hyperborea*. B) Cluster formation of European
1155 Starlings, *Sternus vulgaris*, over Rome (© 2009 STARFLAG project, INFN-CNR).

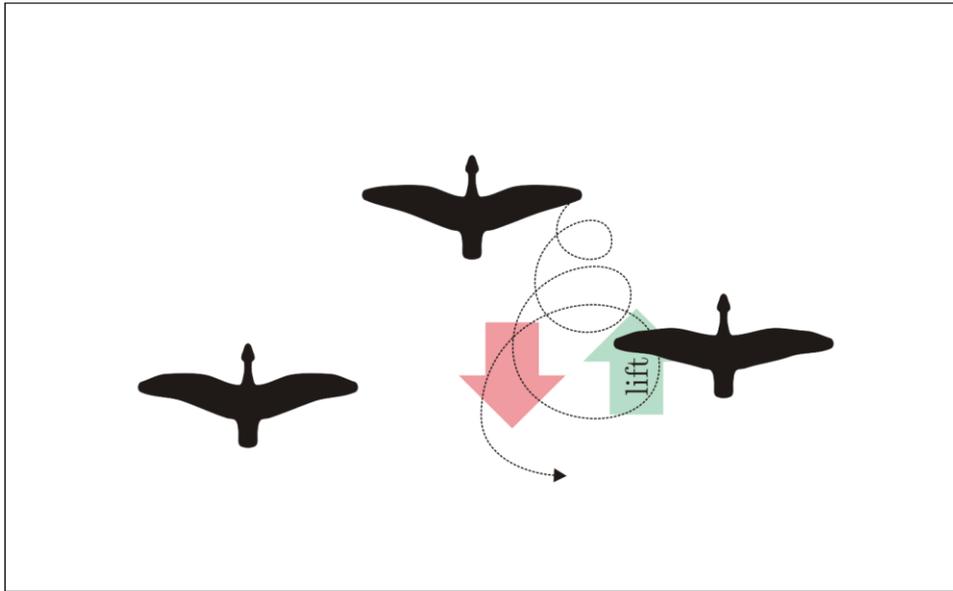


A) Newtonian lift

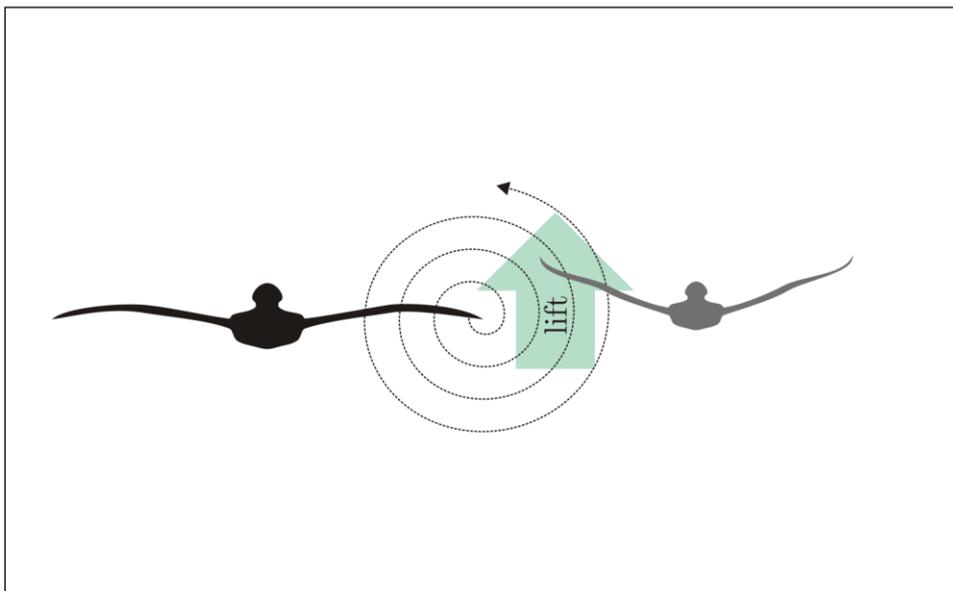


B) Bernoulli lift

1156 **Figure 2.** Lift is generated in two principal ways. A) Air is deflected downward when there is a
 1157 positive angle of attack between a wing and the relative wind. This deflection produces a
 1158 reactive 'Newtonian lift' force. B) When air moves faster over the top of the wing than the
 1159 bottom, a reduced pressure area is created on the top of the wing, generating 'Bernoulli lift'.



A) Top view



B) Front view

1160 **Figure 3.** Air streams off of a bird's wingtip as a horizontal vortex that has a rising and falling
 1161 component. If the wingtip of a following bird were positioned in the rising component of the
 1162 vortex generated by a preceding bird, some of the energy lost by that bird into the tip vortex, as
 1163 a product of generating lift, might be recaptured by a following bird.