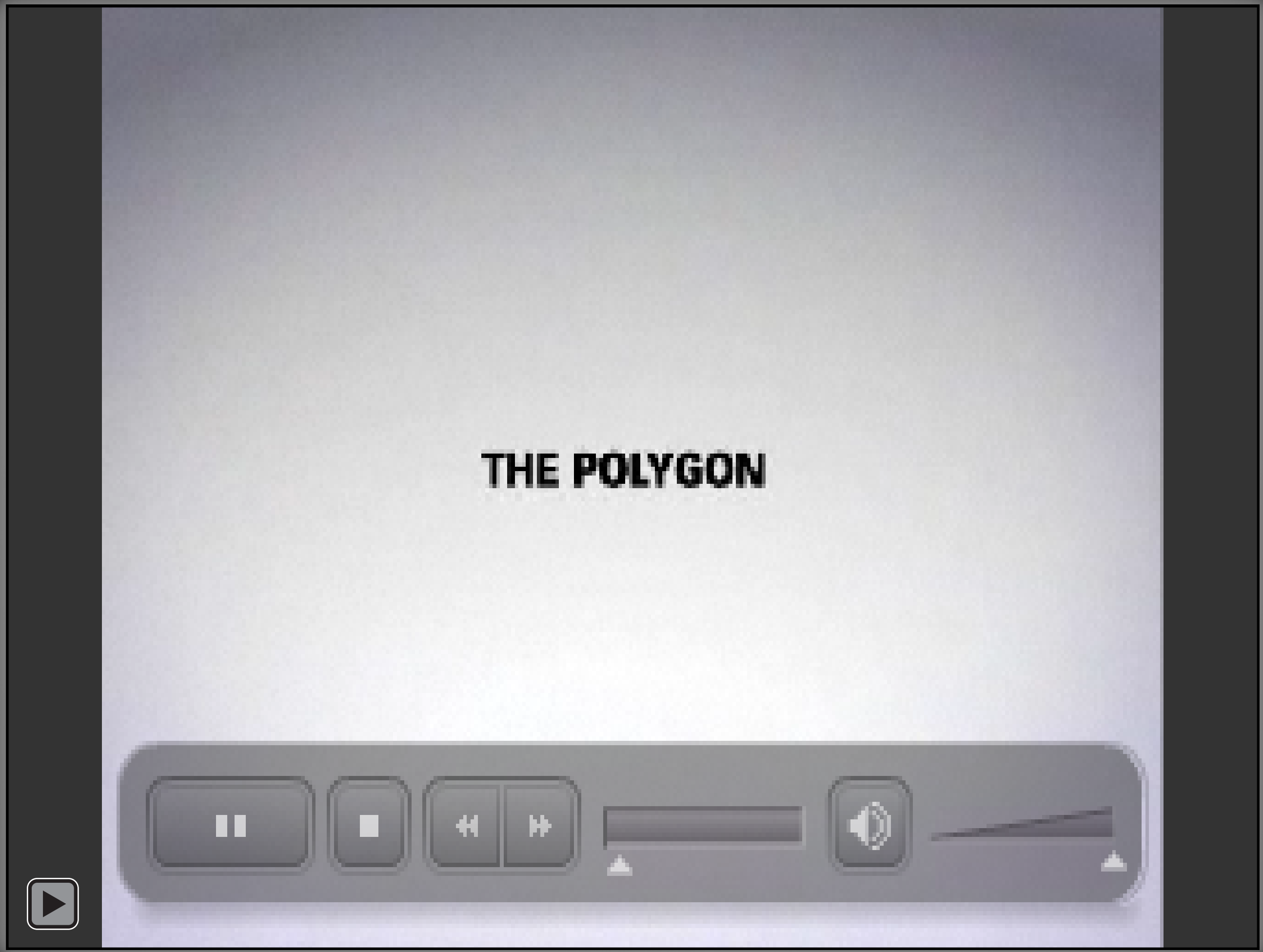
Računalniška grafika

transformacije in
homogene koordinate

točka, vektor

ploskev

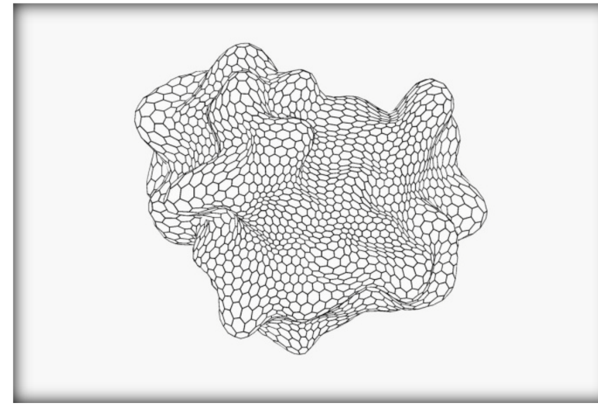


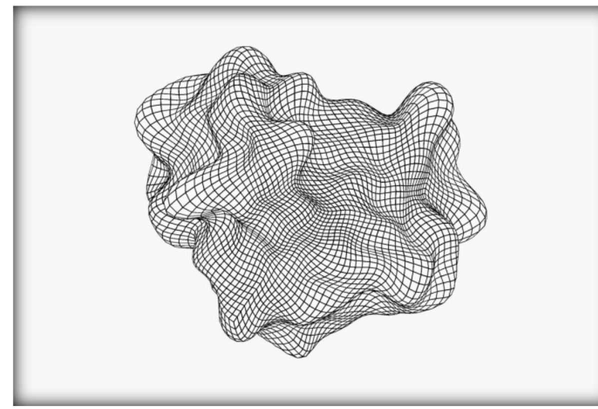
teselacija; angl. *tessellation*

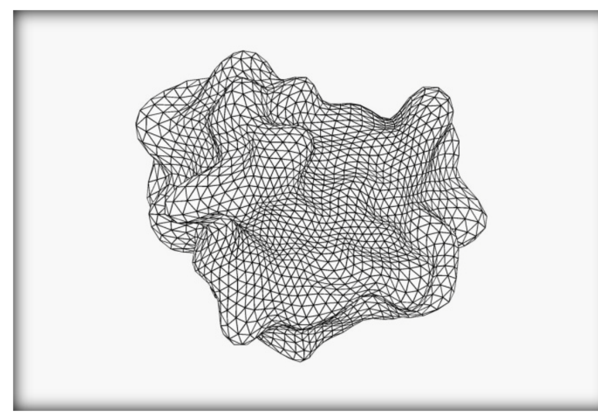
MOZAIČENJE

teselacija

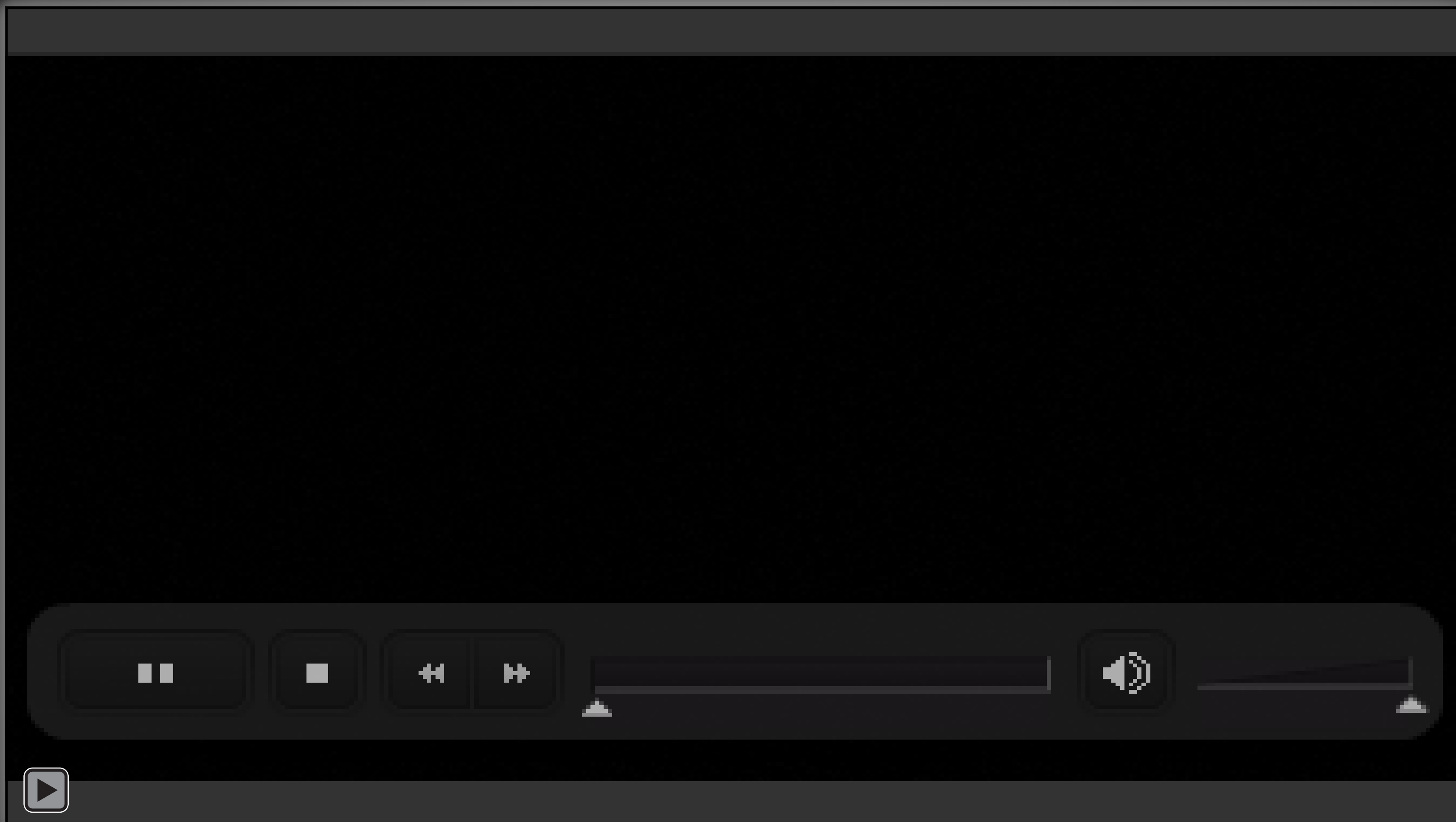
računalniška grafika







teselacija
strojna podpora



“not math again”

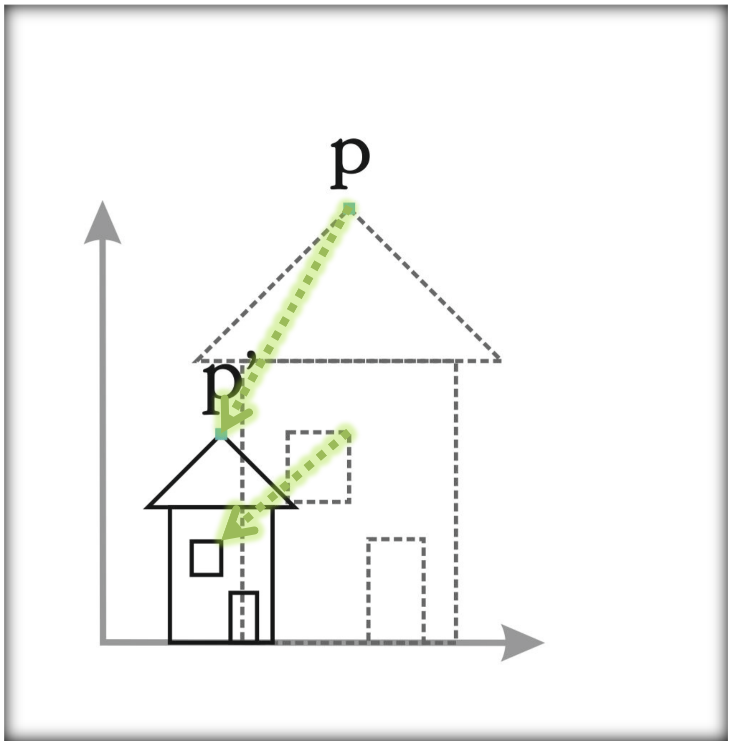
$$p' = f(p)$$

LINEARNE TRANSFORMACIJE

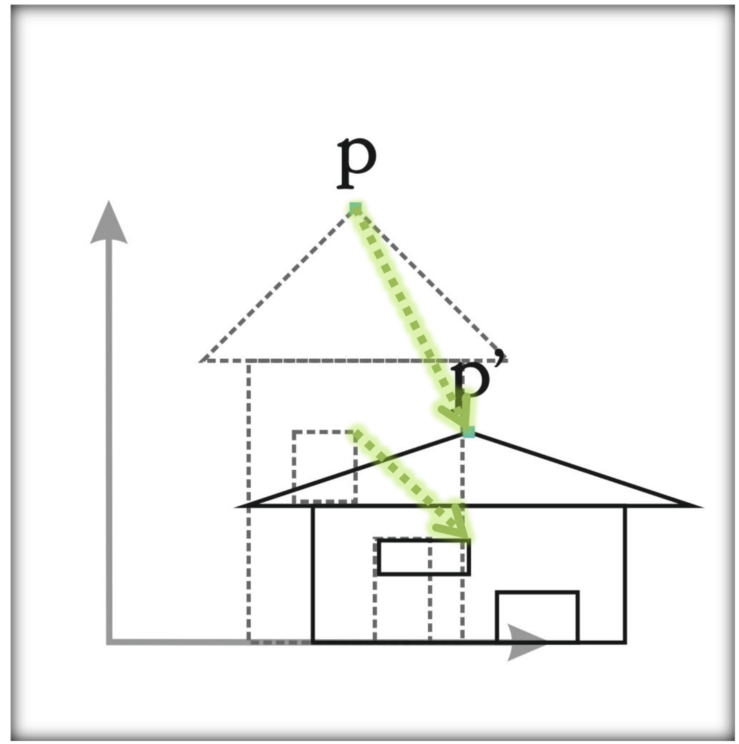
računalniška grafika = matematika
(vektorji, matrike, linearna algebra)

razteg/skaliranje

enakomeren
neenakomeren



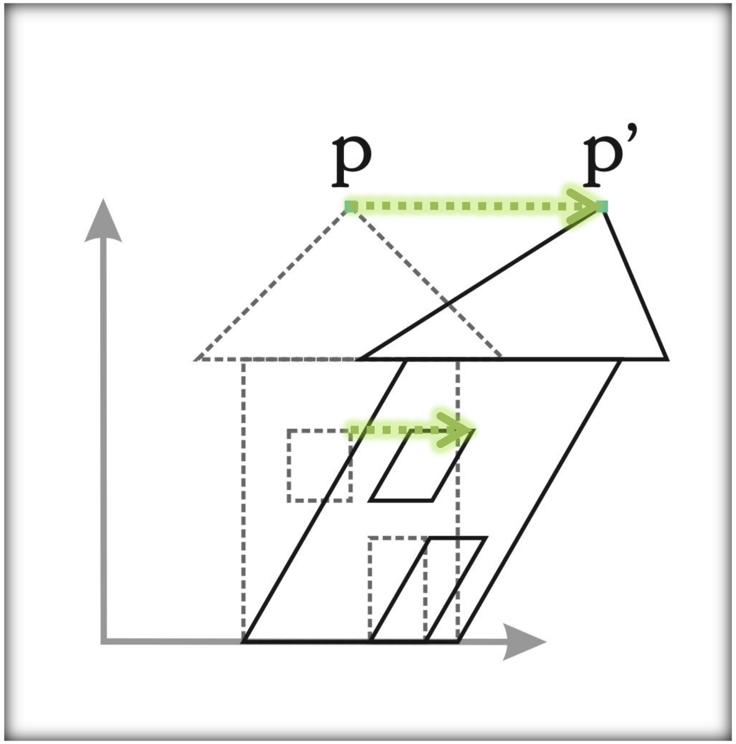
$$\mathbf{p}' = s\mathbf{p}$$



$$p'_x = s_x p_x, \quad p'_y = s_y p_y$$

striženje

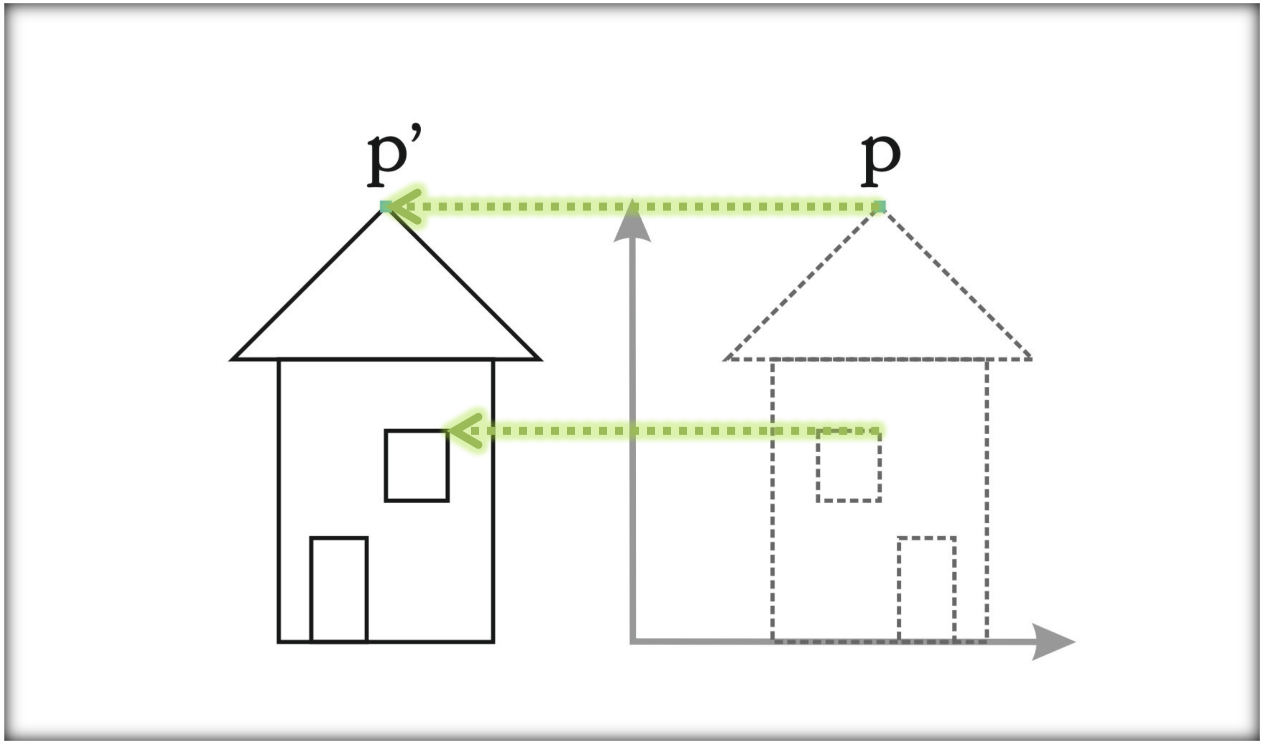
striženje za kot φ



$$p'_x = p_x + z_x p_y,\ p'_y = p_y$$

zrcaljenje

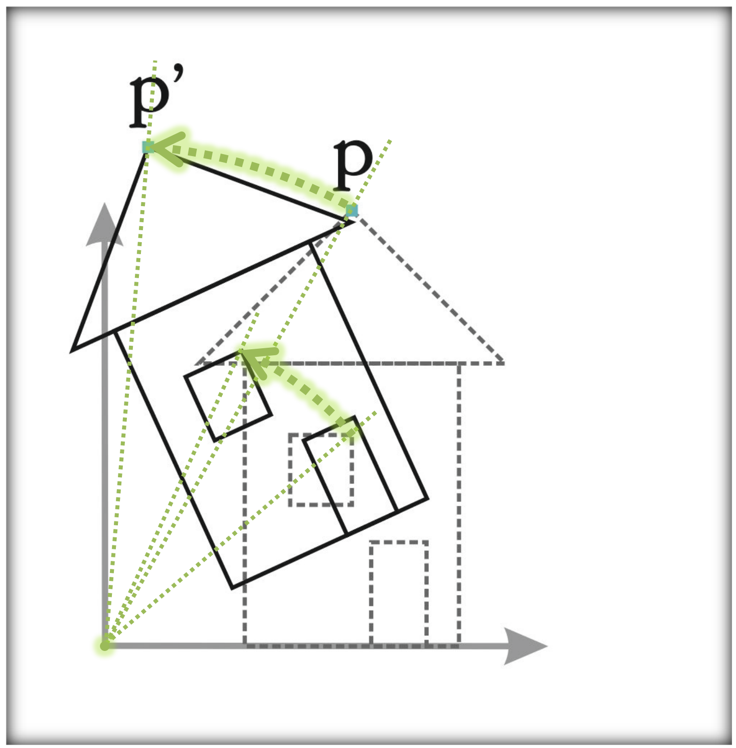
zrcaljenje čez koordinatne osi
zrcaljenje čez os x = y



$$p'_x = -p_x, \; p'_y = p_y$$

vrtenje

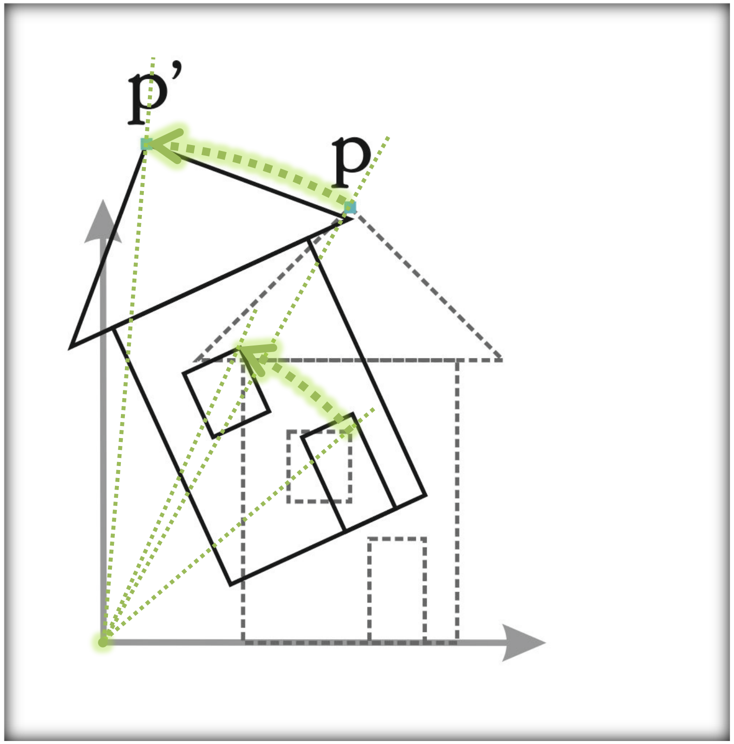
vrtenje okrog izhodišča



$$p'_{\mathsf{x}} = p_{\mathsf{x}} \cos\theta - p_{\mathsf{y}} \sin\theta,\ p'_{\mathsf{x}} = p_{\mathsf{x}} \sin\theta + p_{\mathsf{y}} \cos\theta$$

vrtenje

zapis s polarnimi koordinatami
vrtenje v polarnih koordinatah
adicijski izreki kotnih funkcij
pretvorba v kartezične koordinate



$$p_x = \|\mathbf{p}\| \cos\phi$$
$$p_y = \|\mathbf{p}\| \sin\phi$$

$$p'_x = \|\mathbf{p}\| \cos(\phi + \theta)$$
$$p'_y = \|\mathbf{p}\| \sin(\phi + \theta)$$

$$p'_x = \|\mathbf{p}\| \cos\phi \cos\theta - \|\mathbf{p}\| \sin\phi \sin\theta$$
$$p'_y = \|\mathbf{p}\| \cos\phi \sin\theta + \|\mathbf{p}\| \sin\phi \cos\theta$$

$$p'_x = p_x \cos\theta - p_y \sin\theta$$
$$p'_y = p_x \sin\theta + p_y \cos\theta$$

linearne transformacije

nova točka = matrika * točka
matrika 2x2 za 2D
matrika 3x3 za 3D

$$\mathbf{p}' = \mathbf{M}\mathbf{p}$$

$$\mathbf{S}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$\mathbf{Z}(z_x, z_y) = \begin{bmatrix} 1 & z_x \\ z_y & 1 \end{bmatrix}$$

$$\mathbf{M}_x = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{M}_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{M}_{x=y} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

vrtenje v 3D

os z

$$\mathbf{p}' = \mathrm{R}_z(\theta)\mathbf{p}$$

$$\mathrm{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{p}' = \begin{bmatrix} p_x \cos\theta - p_y \sin\theta \\ p_x \sin\theta + p_y \cos\theta \\ p_z \end{bmatrix}$$

vrtenje v 3D

koordinatne osi

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

vrtenje v 3D

poljubna os

$$\mathbf{p}' = \mathbf{R}(\mathbf{a}, \theta)\mathbf{p}$$

$$c_\theta = \cos\theta,\ s_\theta = \sin\theta$$

$$\mathbf{R}(\mathbf{a},\theta) = \begin{bmatrix} 1 + (a_x{}^2 - 1)(1 - c_\theta) & -a_z s_\theta + a_x a_y (1 - c_\theta) & a_y s_\theta + a_x a_z (1 - c_\theta) \\ a_z s_\theta + a_x a_y (1 - c_\theta) & 1 + (a_y{}^2 - 1)(1 - c_\theta) & -a_x s_\theta + a_y a_z (1 - c_\theta) \\ -a_y s_\theta + a_z a_x (1 - c_\theta) & a_x s_\theta + a_y a_z (1 - c_\theta) & 1 + (a_z{}^2 - 1)(1 - c_\theta) \end{bmatrix}$$

$$\mathbf{R}(\mathbf{e}_z,\theta) = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{R}_z(\theta)$$

$$p' = Mp + t$$

AFINE TRANSFORMACIJE

koordinatni sistem

vektor
točka

$$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z, \mathbf{o}$$

$$\mathbf{v} = v_x\mathbf{e}_x + v_y\mathbf{e}_y + v_z\mathbf{e}_z$$

$$\mathbf{p} = p_x\mathbf{e}_x + p_y\mathbf{e}_y + p_z\mathbf{e}_z + \mathbf{o}$$

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = v_x\mathbf{e}_x + v_y\mathbf{e}_y + v_z\mathbf{e}_z$$

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = p_x\mathbf{e}_x + p_y\mathbf{e}_y + p_z\mathbf{e}_z + \mathbf{o}$$

$$\mathbf{p}_{\mathrm{h}} = \begin{bmatrix} wp_{\mathrm{x}} \\ wp_{\mathrm{y}} \\ wp_{\mathrm{z}} \\ w \end{bmatrix}$$

$$\mathbf{p}_{\mathrm{h}} = wp_{\mathrm{x}}\mathbf{e}_{\mathrm{x}} + wp_{\mathrm{y}}\mathbf{e}_{\mathrm{y}} + wp_{\mathrm{z}}\mathbf{e}_{\mathrm{z}} + w\mathbf{o}$$

HOMOGENE KOORDINATE

homogene koordinate

predstavitev točke
predstavitev vektorja

$$\mathbf{p}_{\mathrm{h}} = \begin{bmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \\ p_{\mathrm{z}} \\ 1 \end{bmatrix}$$

$$\mathbf{v}_{\mathrm{h}} = \begin{bmatrix} v_{\mathrm{x}} \\ v_{\mathrm{y}} \\ v_{\mathrm{z}} \\ 0 \end{bmatrix}$$

$$\mathbf{p}'_{\mathrm{h}} = \mathbf{p}_{\mathrm{h}} + \mathbf{v}_{\mathrm{h}} = \begin{bmatrix} p_{\mathrm{x}} + v_{\mathrm{x}} \\ p_{\mathrm{y}} + v_{\mathrm{y}} \\ p_{\mathrm{z}} + v_{\mathrm{z}} \\ 1 \end{bmatrix}$$

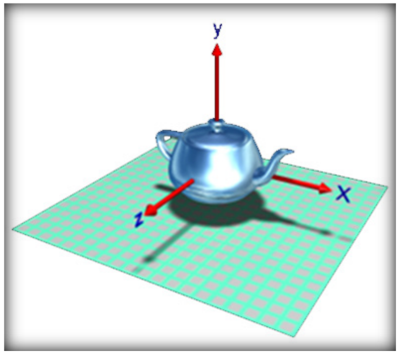
premik

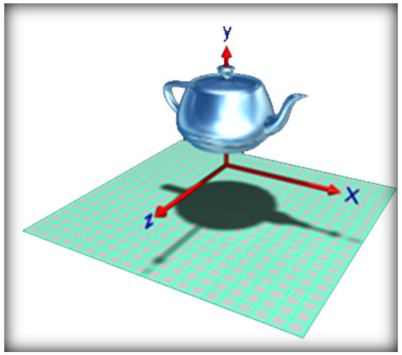
v homogenih koordinatah

$$\mathbf{p}' = \mathbf{T}(\mathbf{t})\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \\ 0 \end{bmatrix} \Rightarrow \mathbf{p}' = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$\mathbf{p}' = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x + t_x \\ p_y + t_y \\ p_z + t_z \\ 1 \end{bmatrix}$$





premik

inverzna operacija

$$\mathrm{T}(\mathrm{t}) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{T}(\mathrm{t})^{-1} = \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

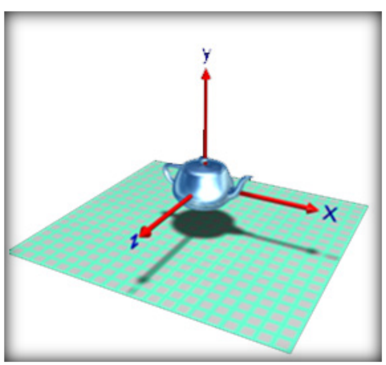
razteg

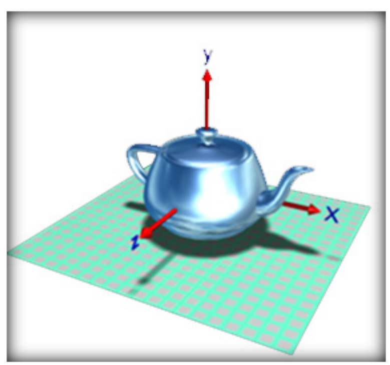
v homogenih koordinatah

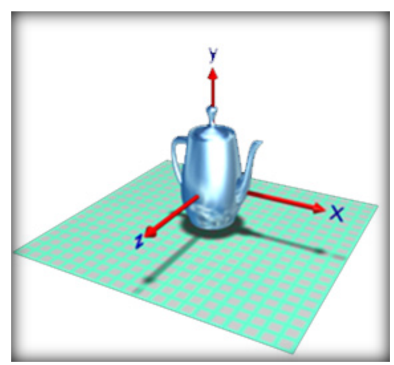
$$\mathbf{p}' = \mathbf{S}(s_x, s_y, s_z)\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \Rightarrow \mathbf{p}' = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$\mathbf{p}' = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} = \begin{bmatrix} s_x p_x \\ s_y p_y \\ s_z p_z \\ 1 \end{bmatrix}$$







razteg

inverzna operacija

$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S(s_x, s_y, s_z)^{-1} = S(1/s_x, 1/s_y, 1/s_z)$$

vrtenje

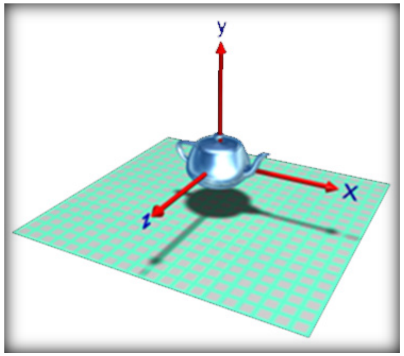
v homogenih koordinatah
koordinatne osi

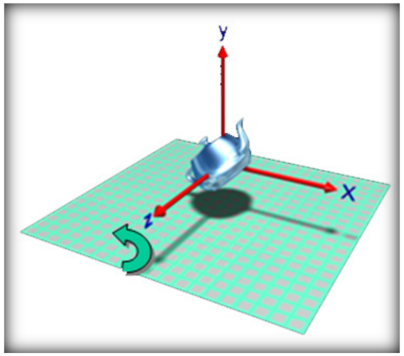
$$p' = R(\theta)p$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





vrtenje

poljubna os

$$\mathrm{p}' = \mathrm{R}(\mathrm{a},\theta)\mathrm{p}$$

$$\mathrm{R}(\mathrm{a},\theta) = \begin{bmatrix} 1+(a_x^{\,2}-1)(1-c_\theta) & -a_z s_\theta + a_x a_y(1-c_\theta) & a_y s_\theta + a_x a_z(1-c_\theta) & 0 \\ a_z s_\theta + a_x a_y(1-c_\theta) & 1+(a_y^{\,2}-1)(1-c_\theta) & -a_x s_\theta + a_y a_z(1-c_\theta) & 0 \\ -a_y s_\theta + a_z a_x(1-c_\theta) & a_x s_\theta + a_y a_z(1-c_\theta) & 1+(a_z^{\,2}-1)(1-c_\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

vrtenje

inverzna operacija
ortogonalnost in lastnosti

$$R(\mathbf{a},\theta)^{-1} = R(\mathbf{a},-\theta) = R(\mathbf{a},\theta)^{T}$$

$$R(\mathbf{a},2\theta) = R(\mathbf{a},\theta)^{2} = R(\mathbf{a},\theta)R(\mathbf{a},\theta)$$

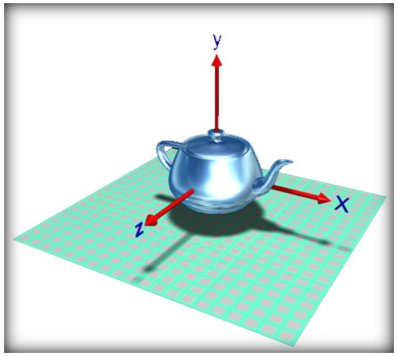
$$R(\mathbf{a},3\theta) = R(\mathbf{a},\theta)^{3} = R(\mathbf{a},\theta)R(\mathbf{a},\theta)R(\mathbf{a},\theta)$$

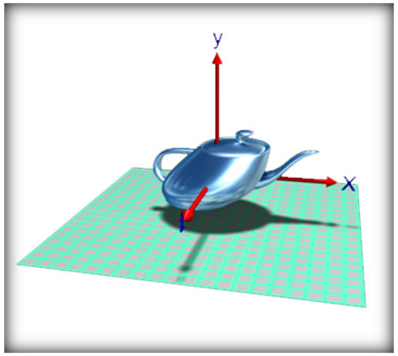
striženje

v homogenih koordinatah

$$\mathrm{p}' = \mathrm{Z}(z_1, \ldots, z_6)\mathrm{p}$$

$$\mathrm{Z}(z_1, \ldots, z_6) = \begin{bmatrix} 1 & z_1 & z_2 & 0 \\ z_3 & 1 & z_4 & 0 \\ z_5 & z_6 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





transformacijska matrika

lastnosti

razteg
vrtenje
striženje
zrcaljenje

premik

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

perspektivna preslikava

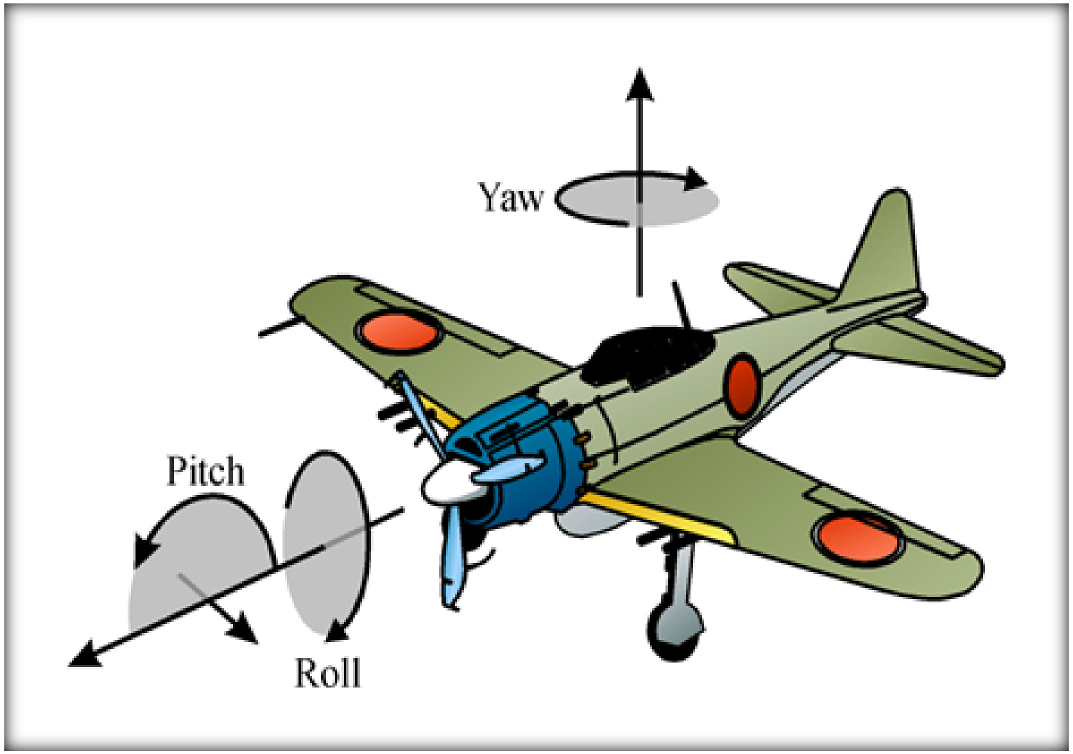
splošna povečava

$$p' = M_3 M_2 M_1 p$$

VERIŽENJE TRANSFORMACIJ

vrtenje v 3D

Eulerjevi koti



$$R_E(\theta_h, \theta_p, \theta_r) = R_z(\theta_r) R_x(\theta_p) R_y(\theta_h)$$

veriženje transformacij

komutativnost
stolpčne matrike
vrstične matrike

$$p' = Mp$$

$$p' = M_3 M_2 M_1 p$$

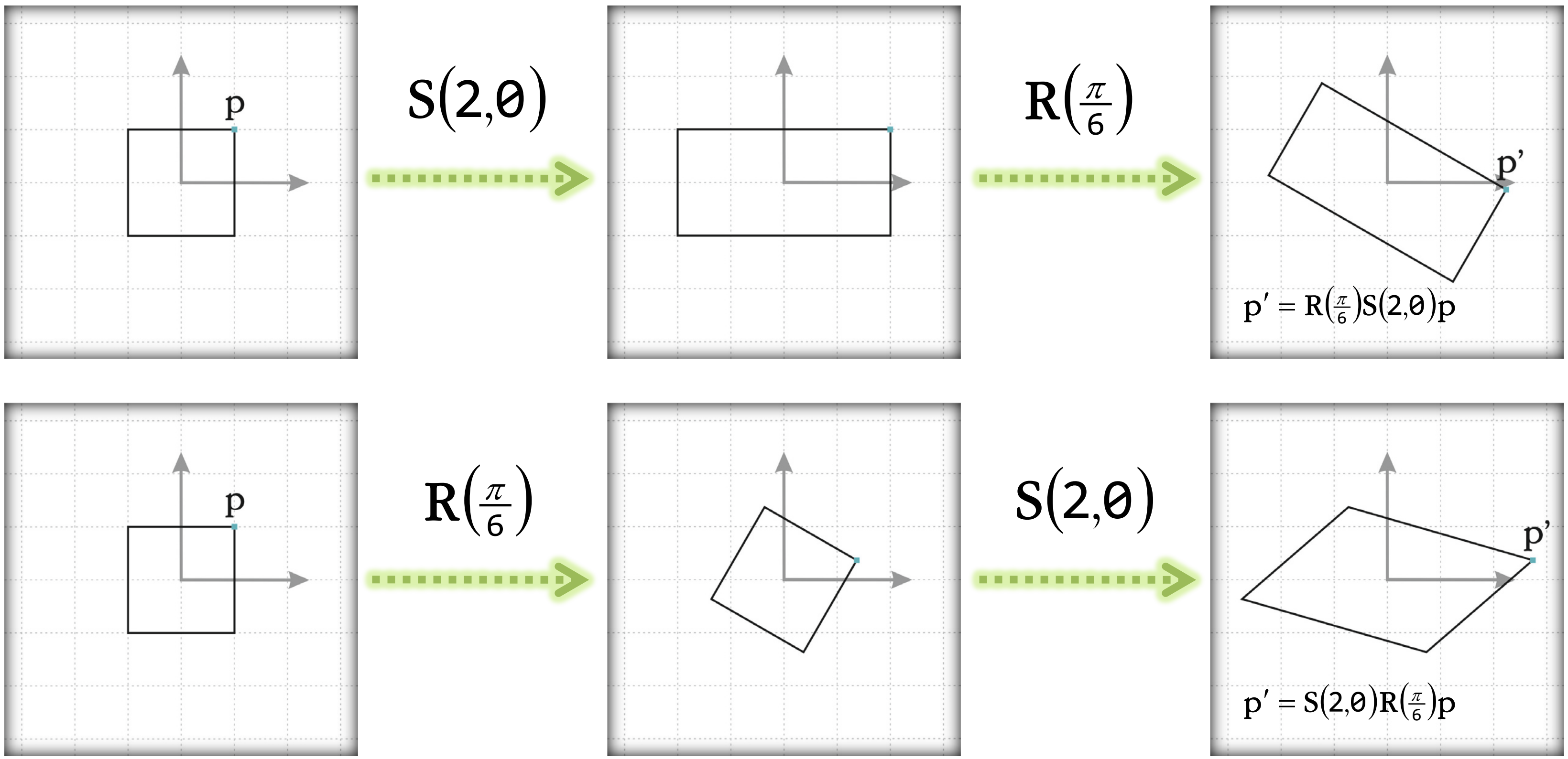
$$M_3(M_2(M_1 p)) = (M_3 M_2) M_1 p = M_3 (M_2 M_1) p = (M_3 M_2 M_1) p$$

$$p'^T = p^T M^T$$

$$p'^T = p^T M_1^T M_2^T M_3^T$$

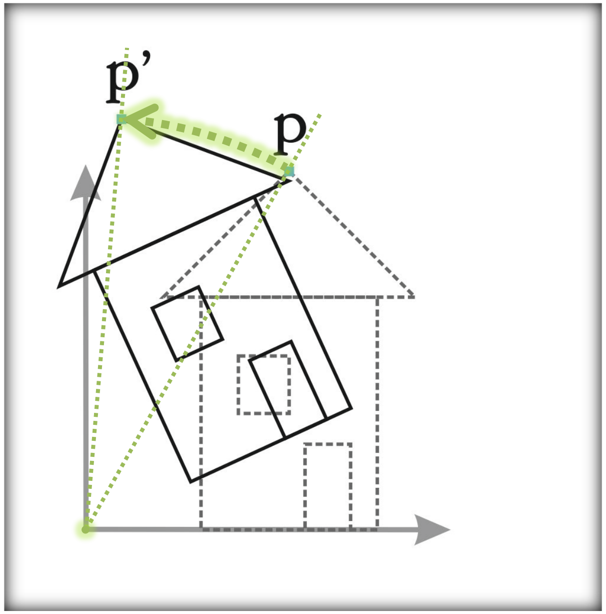
veriženje transformacij

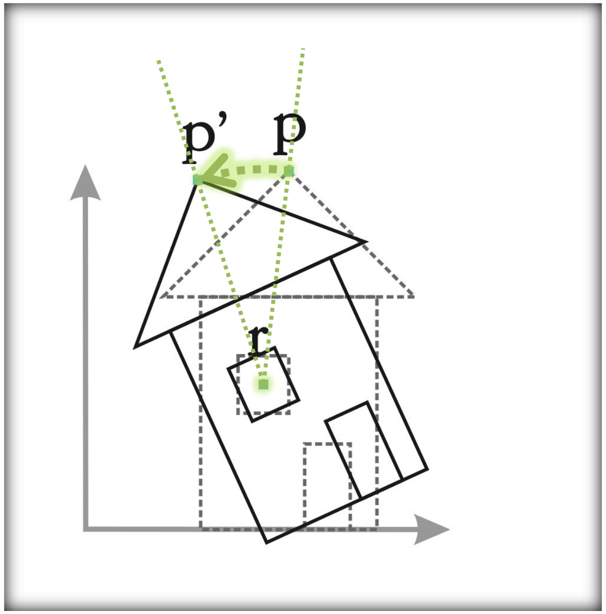
vrstni red operacij je pomemben



veriženje transformacij

vrtenje okrog poljubne točke



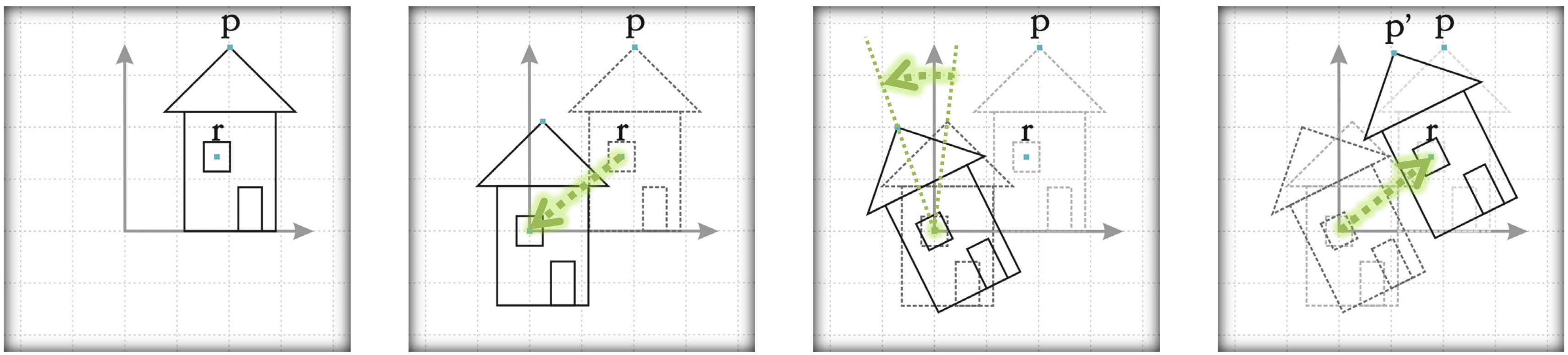


veriženje transformacij

vrtenje okrog poljubne točke

$$\mathrm{p}' = \mathrm{M}_3\mathrm{M}_2\mathrm{M}_1\mathrm{p}$$

$$\mathbf{p}' = \mathbf{T}(\mathbf{r})\mathbf{R}(\theta)\mathbf{T}(-\mathbf{r})\mathbf{p}$$



1. premik v točko

2. vrtenje

3. premik nazaj

TOGE TRANSOFMRACIJE

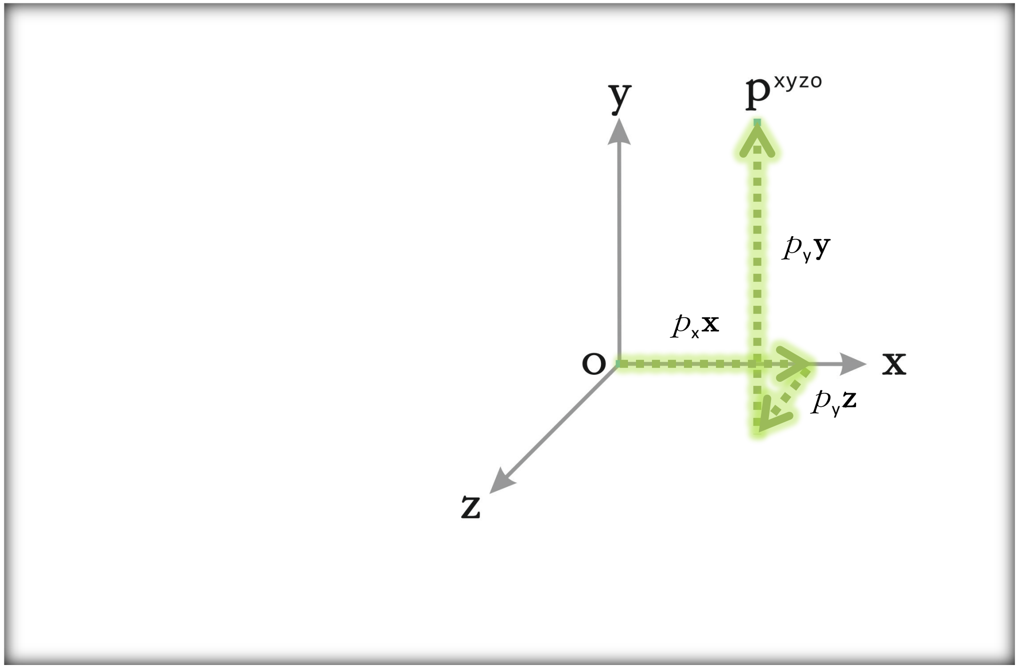
prehodi med

KOORDINATNIMI SISTEMI

točki $\mathbf{p}$, podani v koordinatnem sistemu ($\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, $\mathbf{o}$), poišči koordinate glede na koordinatni sistem ($\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, $\mathbf{q}$)

koordinatni sistemi

prehajanje

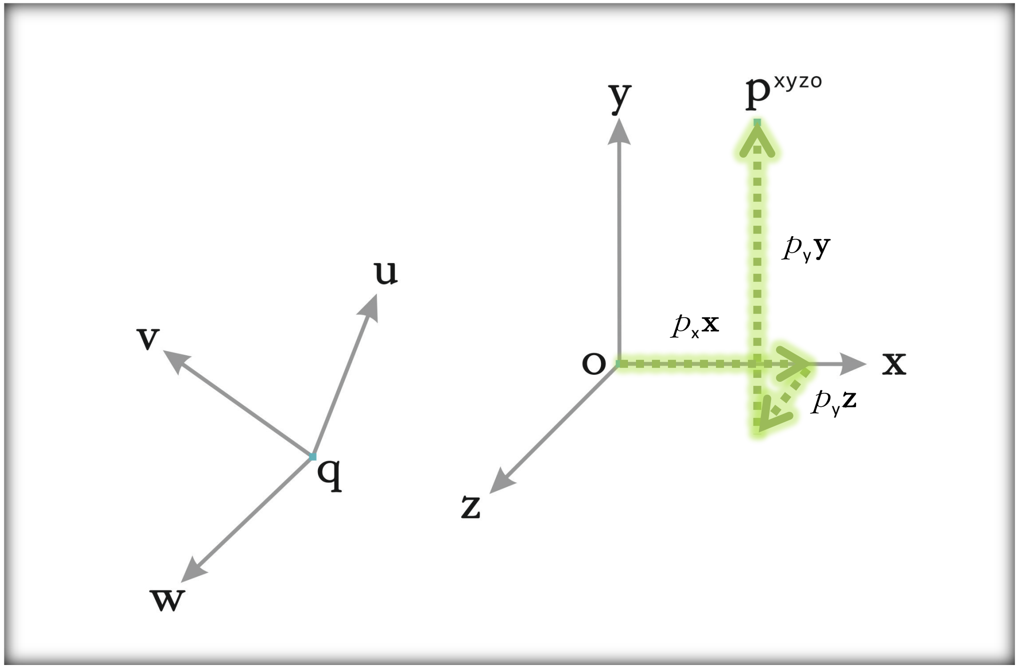


$$\mathbf{p}^{xyzo} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$\mathbf{p}^{xyzo} = p_x\mathbf{x} + p_y\mathbf{y} + p_z\mathbf{z} + \mathbf{o}$$

koordinatni sistemi

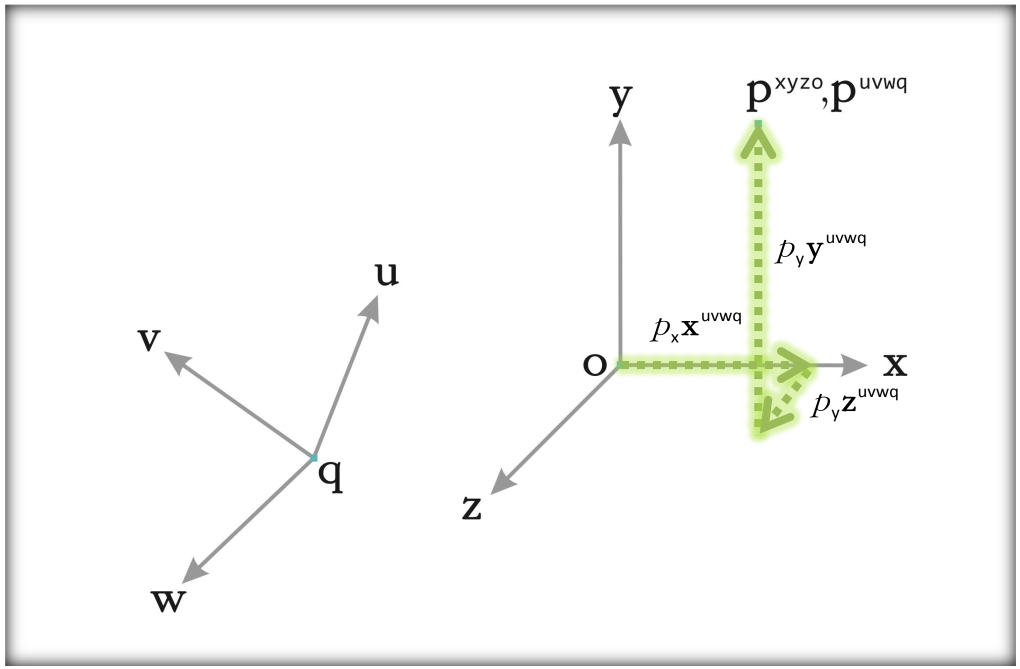
prehajanje



$$\mathbf{x}^{\text{uvwq}} = \begin{bmatrix} x_u \\ x_v \\ x_w \\ 0 \end{bmatrix} \quad \mathbf{y}^{\text{uvwq}} = \begin{bmatrix} y_u \\ y_v \\ y_w \\ 0 \end{bmatrix} \quad \mathbf{z}^{\text{uvwq}} = \begin{bmatrix} z_u \\ z_v \\ z_w \\ 0 \end{bmatrix} \quad \mathbf{o}^{\text{uvwq}} = \begin{bmatrix} o_u \\ o_v \\ o_w \\ 1 \end{bmatrix}$$

koordinatni sistemi

prehajanje



$$\mathbf{p}^{uvwq} = p_x \mathbf{x}^{uvwq} + p_y \mathbf{y}^{uvwq} + p_z \mathbf{z}^{uvwq} + \mathbf{o}^{uvwq}$$

koordinatni sistemi

prehajanje

$$\mathbf{p}^{uvwq} = \begin{bmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{o} \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

koordinatni sistemi

prehajanje

B   T

$$\mathbf{p}^{uvwq} = \begin{bmatrix} x_u & y_u & z_u & o_u \\ x_v & y_v & z_v & o_v \\ x_w & y_w & z_w & o_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

koordinatni sistemi

prehajanje

$$p^{uvwq} = \begin{bmatrix} B & T \\ 0 & 1 \end{bmatrix} p^{xyzo}$$

$$p^{xyzo} = \begin{bmatrix} B^{-1} & -B^{-1}T \\ 0 & 1 \end{bmatrix} p^{uvwq}$$

transformacije

linearne transformacije
zrcaljenje, razteg, striženje, vrtenje

afine transformacije
linearne transformacije, premik

homogene koordinate
predstavitev točke, predstavitev vektorja

ortogonalne transformacije
zrcaljenje, vrtenje, premik

toge transformacije
vrtenje, premik

veriženje transformacij
vrstni red transformacij, vektor kot vrstična matrika

vrtenje
okrog koordinatnih osi, okrog poljubne osi, okrog poljubne točke

prehodi med koordinatnimi sistemi

http://www.cs.brown.edu/exploratories/freeSoftware/repository/edu/brown/cs/exploratories/applets/transformationGame/transformation_game_guide.html

Lengyel, *Mathematics for 3D Game Programming & Computer Graphics*, **ch 3**

Dunn, Parberry, *3D Math Primer for Graphics and Game Development*, **ch 8**

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